3D DISCRETE ELEMENT MODEL OF SILO DISCHARGE

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Abstract: Silos are used by food industry, agriculture, mining and pharmaceutical industries for storing granular materials. The outflow properties of silos depend on the properties of the stored material and on the silo design. For this reason this has long been a subject of interest to both researchers and process engineers. In this paper a new numerical model for modeling silo discharge is proposed. This model was made by Discrete Element Method. We used this method because all of the analytical models are not able to describe the process in non-stationary case but DEM is suitable for describe the whole discharge process. Relying on previous models we established a new numerical model of silo discharge.

Key words: silo, discharge, silo discharge, discrete element method.

INTRODUCTION

Silos are used by food industry, agriculture, mining and pharmaceutical industries for storing granular materials. The outflow properties of silos depend on the properties of the stored material and on the silo design. For this reason this has long been a subject of interest to both researchers and process engineers [14]. The geometry and the surface conditions of the bin and the properties of the stored material define the flow pattern. In general the silos are classified in two different types: mass flow and funnel flow (Figure 1) [10].

In a mass flow silo all of the granular material moves when the silo is discharged, the material that enters the silo first is discharged first (this is a "first in, first out"

behavior). In a funnel flow design the movement of the material is confined to the vertical region in the center of the silo, the material that enters first is discharged last [10]. In case of a funnel flow silo stagnant zones are near the silo walls, where the particles flow slowly or there are stagnant. In extreme cases the silo may not empty completely under the force of gravity.
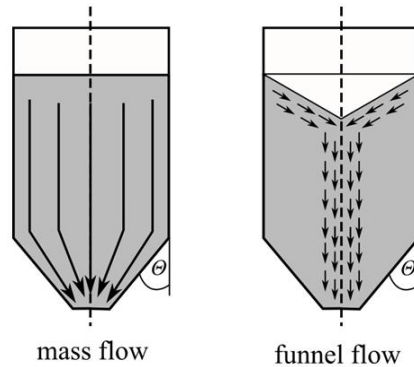

Figure 1. Flow patterns

The flow pattern depends on the properties of the granular material (e.g.: internal friction coefficient) the half angle of the bin and the wall friction coefficient. A lot of studies of granular materials have been described the flow mode of silos. By using the continuum model by Jenike [10], it is possible to define the flow pattern for a given system. The model of Jenike was validated against experimental data to determine a series of design charts (Figure 2). These design charts were verified by others methods, e.g.: DEM [13].
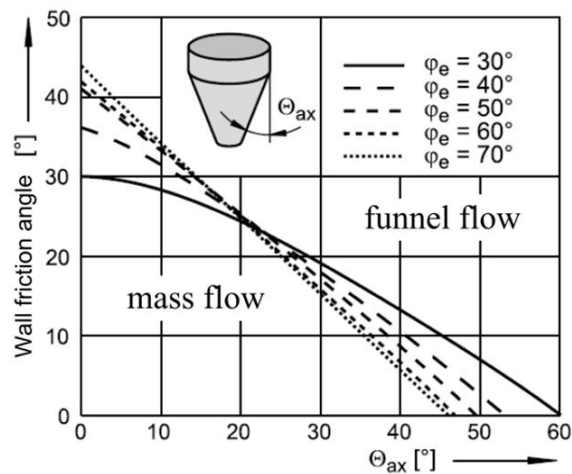

Figure 2. Bin design chart based on theory of Jenike [10]

The discharge rate of granular materials differs from the liquids (Figure 3). In case of fluids the discharge rate and the velocity are changing by the fluid level but in case of granular materials the discharge rate and the flowing velocity is independent of the filling level of silo, therefore constant [16].
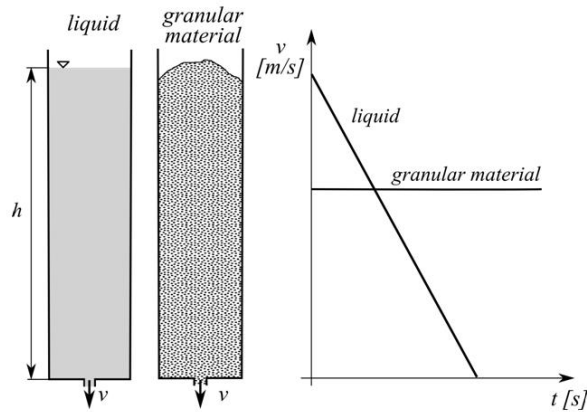

Figure 3. The flowing velocity meanwhile in the filling level

One of the first studies of granular flow in silos was based on a continuum model by Janssen [10]. His analysis showed that the pressure on the bottom of the bin is independent of the filling level. Therefore Janssen's theory predicts that the discharge rate is independent for the filling level. Significant efforts have been made since Janssen's work to determine the discharge rate of a silo using experiments. Thus several theories were forming to determinate the discharge rate of silos:

- For cohesionless granular materials:
  - Hagen model [9],
  - Beverloo model [1],
  - British model [2],
  - Crewdson model[15],
  - Williams model [21],
  - Zanker model [22],
  - Gjacev-Keller model [6]
  - Oldal model [16].
- For cohesive granular materials:
  - Janssen model [10],
  - Johanson model [21],
  - Tomas model [15],
  - Carleton model [3].

From the above the Beverloo's empirical and the Oldal's analytical model can be used in particular circumstances. Other models are empirical and these are unable to explain the physical process causing the phenomena (the constant flowing velocity and constant discharge rate in the same time) and unable to determine the discharge rate. The

Oldal model was developed for the outflow process, based on the assumption that the constant discharge rate of silos is caused by the formation and collapse of arches in the bin. This model explains the constant flowing velocity and the constant discharge rate also [16]:

$$W = \frac{\pi\sqrt{2g}}{6} \cdot \sqrt{\delta} \cdot \rho_h \cdot \left(d - d_p\right)^{\frac{5}{2}}.$$ (1)

where:
W [kg·s^{-1}]   - discharge rate,
δ [-]   - shape coefficient of arch,
ρ_h [kg·m^{-3}] - density of the bulk material,
g [m·s^{-2}]   - gravitational acceleration,
d [m]   - outlet size,
d_p [m]   - particle size.

The difference between the empirical models and Oldal's model is that this model doesn't use empirical constants for describing the outflow. The δ constant is derived from the physical description of the discharge process. This model predicts the constant discharge rate and also the non-constant velocity distribution. Still this analytical model is unable to determine the discharge rate in the case of a mass flow silo.

In this paper a new numerical model for modeling the silo discharge is proposed. This model was made by Discrete Element Method. Our work is based on laboratory outflow experiments and simulations. We were making simulations and outflow measurements with wheat. To validate the numerical model, we created model silos with different cone angles. The aim of our work was a creation of a numerical model for silo discharge which can explain the constant flowing velocity and the constant discharge rate, therefore the phenomenon and to validate that the micromechanical parameters of wheat which validated for mixed flow dryer [12] are suitable for modeling silo discharge.

MATERIAL AND METHODS

The Discrete Element Method (DEM) is a numerical technique for modeling the static and dynamic mechanical properties of the granular materials. The method is based on the simulation of the motion of granular materials as separate particles and involves the movement of all particles and detection their collision with other particles and with their environment [4]. DEM is commonly used in many fields of engineering such as in the pharmaceutical, mining, food industries or agriculture. In agriculture this is commonly used to define the behavior and motion of granular materials in dryers or in silos and hoppers, such as flow patterns, segregations, and discharge rate. The method is continually developed and the experimental studies are also now gone on.

When DEM is used then the equations of motion on single particles are solved by a simulation circle. To describe the movement of the single particles Newton's second law of motion and the general rotational dynamics equation are repeatedly used. The contact

forces and the moments are calculated based on the displacement of the particles in every time steps:

$$\underline{\underline{M}}^{i}(t) \cdot \underline{a}^{i}(t) = \underline{f}^{i}(t, \underline{u}^{i}(t), \underline{v}^{i}(t)). \tag{2}$$

where:
$\underline{M}$ [kg, kg·m^2] - block diagonal matrix of mass and inertia,
t [s]   - time,
$\underline{a}$ [m·s^{-2}, 1·s^{-2}] - acceleration vector,
$\underline{f}$ [N, Nm]   - load vector,
$\underline{u}$ [m]   - position vector,
$\underline{v}$ [m·s^{-1}, 1·s^{-1}] - velocity vector,
i [-]   - index that is assumed the number of particles.

The equation of the whole granular material:

$$\underline{\underline{M}}(t) \cdot \underline{a}(t) = \underline{f}(t, \underline{u}(t), \underline{v}(t)). \tag{3}$$

The behavior of the particles and the interaction between particles depend on the geometry and the micromechanical properties of the particles and their contacts [17]. The main problem is the determination of the micromechanical parameters (the calibration of the model). Because of the possibilities of errors in the simulations, we always have to compare the simulations with experimental results.

DEM has been used by different authors to study the outflow properties of silos such as pressure distribution [7], [8] flow patterns [13], [8], flow velocity [8], and segregation processes [13].

All simulations were undertaken using EDEM Academic 2.4 (2012) discrete element software. To modeling the outflow of wheat was used the Hertz-Mindlin no slip contact model with the following micromechanical parameters for the description of the interaction between particles:

- Poisson's ratio (v): defined as the ratio of transverse contraction strain to longitudinal extension strain in the direction of stretching force.
- Shear modulus (G): defined as the ratio of shear stress to the shear strain, where the shear stress is the components of stress at a point that act parallel to the plane in which they lie and shear strain is the components of a strain at a point that produce changes in shape of a body without a volumetric change.
- Density (ρ): defined as the "weight" per unit volume.
- Coefficient of restitution (C_r): the ratio of speed of separation to speed of approach in a collision.
- Coefficient of static friction (μ_0).
- Coefficient of rolling friction (μ_r).

The Hertz-Mindlin no slip contact model uses a spring-dashpot model to describe interaction of the particles. This contact model is elastic and non-linear and takes into account viscous and frictional damping. This contact model was used to simulate particle-particle and particle-wall contacts [20]. The micromechanical parameters have

an influence on normal- and tangential forces and moment between the interacting particles. These micromechanical parameters and the particle model for wheat were determined in a mixed flow dryer by [12].

Table 1. Micromechanical parameters of wheat by [12]

Material	v [-]	G [MPa]	ρ [kg·m⁻³]	C_{rw} [-]	C_{rs} [-]	μ_{0w} [-]	μ_{0s} [-]	μ_{rw} [m]	μ_{rs} [m]
Wheat	0,4	$3,58 \cdot 10^8$	1460	0,5	0,6	0,3	0,25	0,01	0,01
Steel	0,3	$8 \cdot 10^{10}$	7500	0,5	-	0,25	-	0,01	0,01

The particle model has been created as the clump of three spheres, having radiuses 3 mm and 2,5 mm respectively. The distance between the centers of the spheres on the edges was 2 mm (Figure 4).

a.   b.
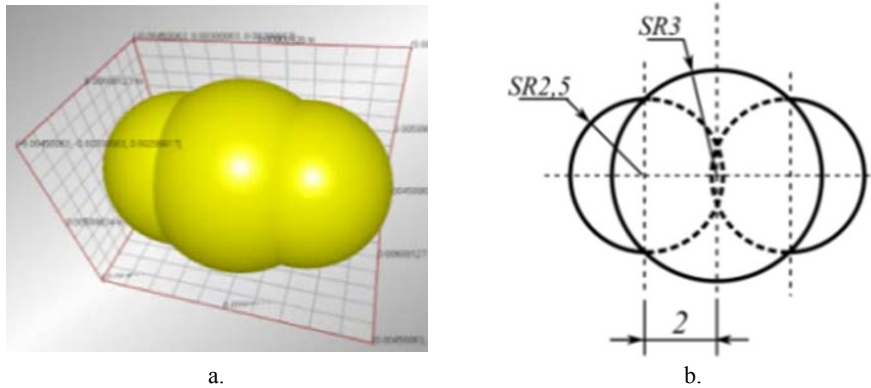
Figure 4. Particle model of a grain of wheat [12]

We used in present wok two groups of geometrical models. In case of first group the geometrical model was a cylinder with a diameter of 200 mm and conical bin with outlet diameter of 60 mm and the cone half angle 30° and 60°. In case of second group the geometrical model was a cylinder with a diameter of 105 mm and conical bin with outlet diameter of 35 mm and the cone half angle 30° and 60°. In all cases a virtual volume was created, which was the same shape and in the same position with the geometrical models.

In the simulation process the first step was the generation of all of the particles, these were generated randomly in the virtual volume. The generated particles were allowed to fall under gravity; this step was the filling of silo. During the filling process the outlet of the silo was closed. The next step was the emptying of the silo. When the particles reach a static state then the outlet of the silo was opened and all of the particles were discharged. To the end of the filling process the bulk reaches a static state (his energy of motion is about zero). On the Figure 5 the particles were painted according to their vertical velocity. Vertical velocity of the blue particles is minimal and the vertical velocity of red particles is maximal.
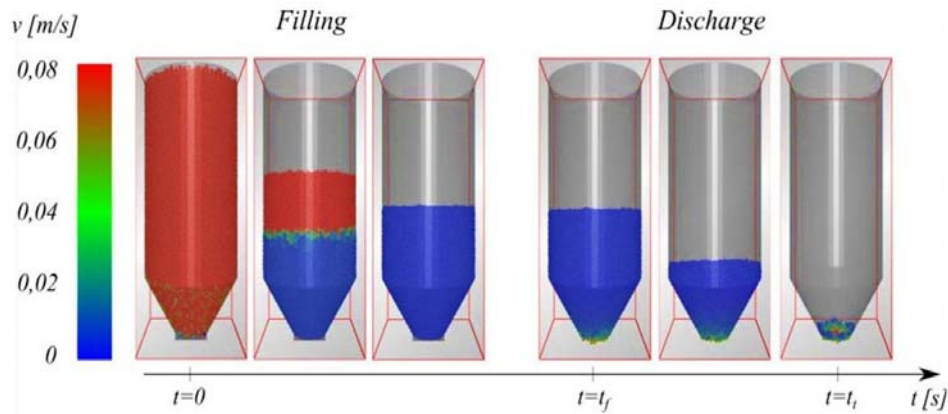
Figure 5. The steps of simulation process

To validate the numerical model, we were creating model silos with same dimensions with the geometrical models. The mass of the discharged wheat was measured by three load cells. The measurements were repeated five times in case of every model silo. The goal of the measurements was the determination of the amount of outflowing wheat as a function in time. The mass change functions were determined by also the simulations and these were compared with the measurement results. The aim of the experiments was the validation of DEM model.

RESULTS AND DISCUSSION

Based on the results of experiments the mass of the discharged material in function of time can see on Chart 1/a. As we expected the mass-change functions are linear in all cases. This means, that the discharge rate is constant, since this is the slope of the linear. Consequently the discharge rate is independent of the filling level of silo.
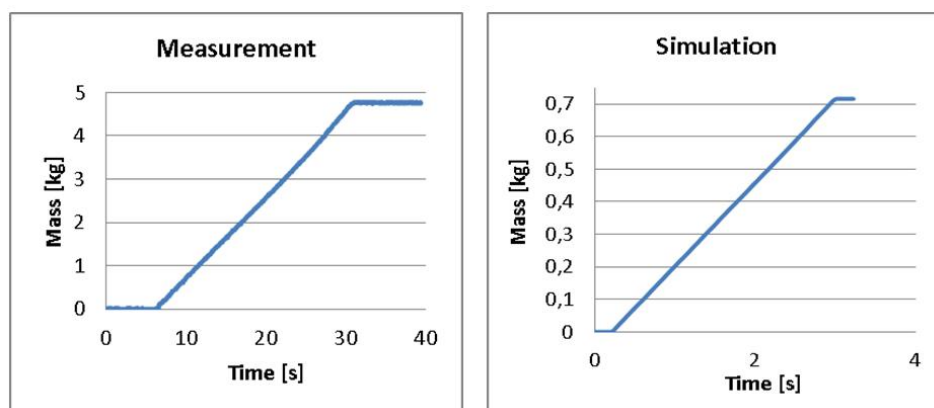

Chart 1. Measured and simulated mass-time values

Similarly to the experiments we determined the mass-change functions based on the simulation results. In case of all simulation the mass-change functions are also linear (Chart 1/b). This means that the new numerical model is suitable for right modeling the physical phenomenon.

Based on the mass-times values the discharge rate was determined in all cases. Then the simulation results were compared with measurements. The difference between our model and measurements is less than 5% in case of a silo with 60 mm outlet diameter by both bins.
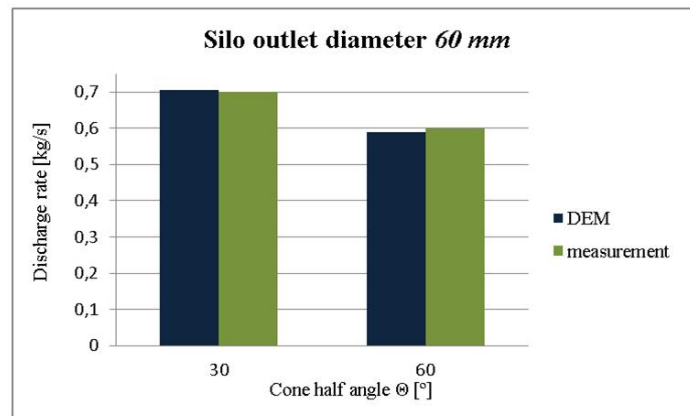

Chart 2. Discharge rates in case of a silo with 60 mm outlet diameter

In case of *35 mm* outlet diameter the difference of the discharge rates is larger by both bins (between 10 and 15%) (Figure 7). The reason of the larger difference is the particle model. The particle model of [12] is a slightly bigger as a real grain of wheat, but this is suitable for determine the velocity distribution in a mixed flow dryer and also suitable for the discharge simulation in case of a silo with 60 mm outlet diameter (the computational requirement can reduce with this particle model).
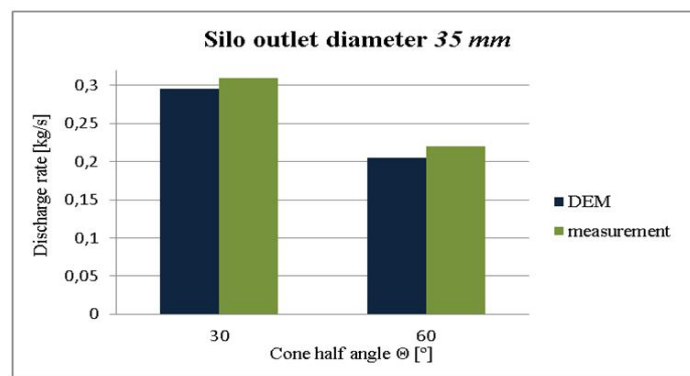

Chart 3. Discharge rates in case of a silo with 35 mm outlet diameter

However this particle model is not suitable for the silo model with outlet diameter 35 mm, because the bigger wheat particles caught on each other above the outlet. For this reason the discharge rate is less than the real. We had to create a new particle model, which is a better likeness the real grain of wheat. The new particle model has been created well as the clump of three spheres, having radiuses 1,5 mm and 1,25 mm respectively. The distance between the center of the spheres on the edges was 1,5 mm (Figure 8).
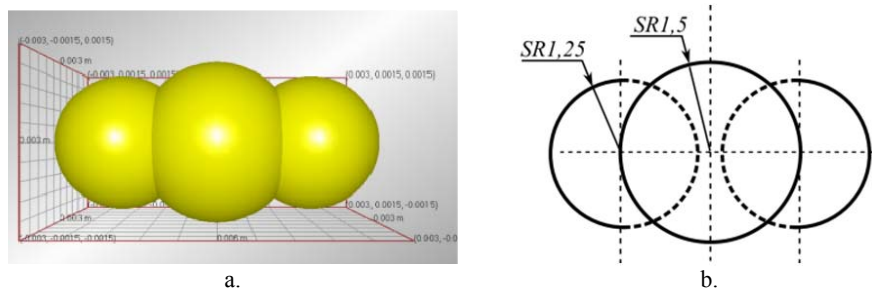

a.   b.

Figure 6. The new particle model of wheat

To this new particle model we had to define also the adequate density of particles:

Table 2. Micromechanical parameters to new particle model

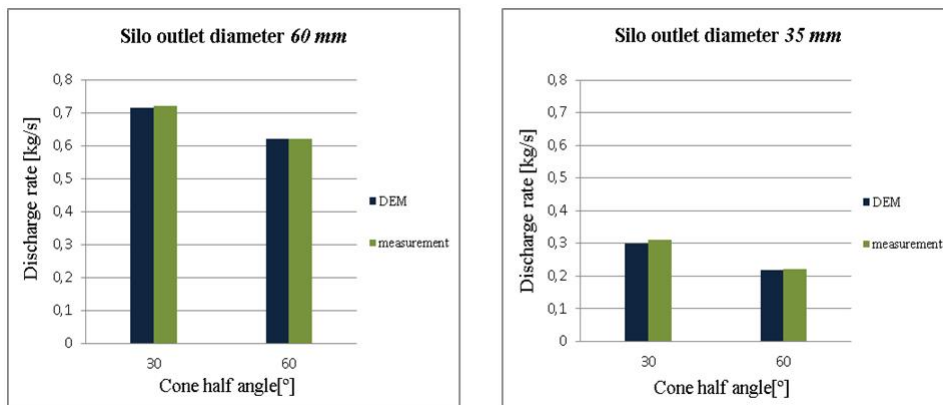| Material	v [-]	G [MPa]	ρ [kg·m⁻³]	C_{rw} [-]	C_{rs} [-]	μ_{0w} [-]	μ_{0s} [-]	μ_{rw} [m]	μ_{rs} [m]
Wheat	0,4	$3,58·10^8$	1430	0,5	0,6	0,3	0,25	0,01	0,01
Steel	0,3	$8·10^{10}$	7500	0,5	-	0,25	-	0,01	0,01

Chart 4. The discharge rates with the new particle model

With the new particle model and micromechanical parameters the difference between experiments and simulations is less than 5% in all cases. So the discharge of wheat can be on a suitable manner modeled.

CONCLUSIONS

We created a new numerical model for silo discharge of wheat which is suitable for description the outflow process with an adequate accuracy. With the numerical model the whole discharge process can be described, still in case of an emptying bin. One of the analytical models is not able to describe the process in this non-stationary case.

The particle model and the micromechanical parameters of wheat which are validated for a mixed flow dryer are suitable for modeling also the silo discharge. With these parameters can reduce the computational requirements because this particle model is a slightly bigger as a real grain of wheat. However we can't use these parameters in all cases because if the outlet diameter of the silo less than 35 mm than the particles caught on each other. For this reason we made a new particle model and we determined for this new micromechanical parameters. We validated these parameters with outflow experiments.

Consequently if a discrete element model for wheat is created in which the flowing dimensions are 35 mm then it is necessary to use our new particle model with the adequate micromechanical parameters.

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3D MODEL PRAŽNJENJA SILOSA KORIŠĆENJEM METODA DISKRETNIH ELEMENATA

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Sažetak: Silosi se koriste u prehrambenoj industriji, poljoprivredi, rudarstvu i farmaceutskoj industriji za skladištenje granulastih materijala. Pražnjenje silosa zavisi od osobina skladištenog materijala i konstrukcije silosa. Iz ovog razloga je pražnjenje uvek bilo značajno kako za istraživače, tako i za procesne inženjere. U ovom radu je predložen novi numerički model za projektovanje pražnjenja silosa. Ovaj model je napravljen korišćenjem Metoda Diskretnih Elemenata. Koristili smo ovaj metod zato što svi analitički modeli nisu u stanju da opišu proces u nestacionarnom slučaju, a MDE je

pogodan za opisivanje kompletnog procesa pražnjenja silosa. Oslanjajući se na prethodni model uspeli smo da uspostavimo novi numerički model pražnjenja silosa.

Ključne reči: *silos, pražnjenje, pražnjenje silosa, metod diskretnih elemenata.*