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MATHEMATICAL MODELING AND USE IN ACOUSTICS APPLICATIONS IN AGRICULTURE

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Abstract: The article presents a discussion and a comparison of modeling methods for acoustic sources, it compares performance models and presents possible applications.

The mathematical models are presented and compared: the natural and theoretical model that is based on the theoretical and empirical model of acoustic wave equation of noise spatial variation. The models are compared in terms of sound intensity level (which is calculated by exact formulas from the wave function solution of the acoustic waves equation) and noise levels provided by the theoretical and empirical model. It is shown that in most cases, both are models whose results must be calibrated in order to give results matching reality. Finally we compare the field application of each of the two models.

Key words: *acoustics theoretical model, theoretical-empirical model, applications, comparisons*

INTRODUCTION

Sound waves are acoustic waves with conventional frequencies between 16 and 20,000 Hz.

Purely theoretical and applicative approach to distribution problems and their effects on acoustic waves (sound or noise) in the environment, which I started a few years ago [1]. The theoretical and empirical approach to sound intensity distribution in space is often a surprise to theorists (mathematicians or physicists). It is surprising to see that the fundamentals of complex software used for creating noise maps [2, 3, 4], consist of simple functions, even elementary, with which one gets very close to the true

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distribution of environmental noise. Theorists expectation was that the foundation of these programs to use the wave equation and by calculating its solution for a given source distribution and a specified boundary of the domain, then build sound intensity level distribution in the target area. Surprisingly at the end of the comparison it is found that the two procedures lead to convergent results regarding the distribution of sound intensity level (theoretical result) and the noise levels from the source (result of the theoretical and empirical model). This convergence strengthens that the two survey instruments propagation of sound waves in space are trustworthy.

FUNDAMENTALS

In order be more clear, the definitions with which we operate must be specified. First, according to [5], an acoustic wave means an elastic wave of small amplitude. The same source defines:

- sound intensity level (L_I), is 10 times the natural logarithm of the ratio of acoustic sound intensity and sound intensity reference, equal to 10-12 $W \cdot m^{-2}$ (the unit is decibels, denoted dB);
- sound pressure level (L), being 20 times the logarithm of the ratio between the considered effective sound pressure sound and reference sound pressure p_0 = $2 \cdot 10^{-5}$ $N \cdot m^{-2}$ (unit dB);
- sound power level (L_p, L_W), being 10 times the logarithm of the ratio between acoustic power radiated by a source and reference power wich is 10-13 W (unit of measurement, dB).

From the engineering point of view, it is expressed for example, in accordance with the standard [6] IEC 61672-1, Section 3.9, "Definitions", according to which equivalent continuous sound level (*LAT*) is 20 times the base 10 logarithm of the ratio between the RMS value of frequency weighted sound pressure during a period of time and reference sound pressure (20 μPa). However, in current use, mainly for historical reasons *LAT* is used as *Leq*.

As shown, the theoretical-empirical mathematical model makes a sound frequency filtering by curve "*A*" frequency weighting, the curve being constructed to protect workers who are exposed to danger at workplace, [7].

THEORETICAL MODEL

A punctual sound source is considered that produce spherical acoustic waves in a boundless medium and without obstacles around. We chose spherical waves because they are the most basic solutions of the wave equation showing attenuation with distance (cylindrical waves are not physically as close to reality as the spherical). The next wave equation is considered:

$$\Delta\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0 \qquad (1)$$

where ψ is the wave function, c is the speed of sound in air at normal physical conditions and Δ is the Laplace operator. The spherical shape solution is:

$$\psi(t,r)=\frac{\psi_0}{r}\cos\left(\omega t-kr+\phi\right) \tag{2}$$

satisfies the wave equation (1), if the following conditions are met:

$$v_f=\frac{\omega}{k}=\frac{\lambda}{T}=c \tag{3}$$

where v is the wave frequency, ω is the angular frequency of the wave, k is the wave number (with dimension of length to the -1 power), λ is the wavelength, T is the wave period and ψ_0 is a constant. It is noted that we consider a monochromatic wave (a single frequency). The constant, which not many authors comment, must have a dimension length to the power of two. The variable r is the distance from the current point to the point where the source is located in space:

$$r(x,y,z)=\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2} \tag{4}$$

Accepting a solution of type (1) for monochromatic point sources and considering ρ as the air density, we obtain the kinetic energy of the wave:

$$w_c=\frac{1}{2}\rho\left(\frac{\partial\Psi}{\partial t}\right)^2 \tag{5}$$

and the potential energy of the same wave is:

$$w_p=\frac{1}{2}\rho c^2\left(\nabla\Psi\cdot\nabla\Psi\right) \tag{6}$$

Then the total energy of the wave is:

$$w=w_c+w_p \tag{7}$$

The square root average value of energy in a time interval is:

$$W=\langle w\rangle=\sqrt{\frac{1}{T}\int_0^T w(t)^2\,dt} \tag{8}$$

and the energy intensity of the acoustic wave energy flux passing through unit area in the direction normal to this surface is:

$$I_s = W \cdot c \tag{9}$$

Sound intensity level is given by:

$$N_s = 10\lg \frac{I_s}{I_{s0}} \tag{10}$$

where I_s is the magnitude of the energy intensity and I_{s0}= 10^{-12} W·m^{-2} is the reference magnitude of the energy intensity and lg is the logarithm in base 10.

According to [8], the effective acoustic pressure is defined by the relation:

$$p_{ef} = \sqrt{I_s \rho c} \tag{11}$$

and accordingly, the reference pressure:

$$p_0 = \sqrt{I_0 \rho c} \tag{12}$$

Then, it is easily deduced that:

$$L = 20\lg \frac{p_{ef}}{p_0} = 20\lg \left(\frac{I_s \rho c}{I_0 \rho c} \right)^{\frac{1}{2}} = 10\lg \frac{I_s}{I_0} = N_s = L_I \tag{13}$$

A similar calculation shows that L_{eq}=N_s:

$$\begin{aligned} L_{eq} &= 10\lg \frac{\sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \left(p_{ef} \right)^2 dt}}{p_{ref}} = 10\lg \frac{\sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \left(I_s(t) \right)^2 \rho^2 c^2 dt}}{p_{ref}} = \\ &= 10\lg 10\lg \frac{\rho c \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} W^2 c^2 dt}}{p_{ref}} \frac{\rho c \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \left(I_s(t) \right)^2 dt}}{p_{ref}} = \\ &= 10\lg \frac{\rho c \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} W^2 c^2 dt}}{p_{ref}} = 10\lg \frac{\rho c I_s}{\rho c I_{s0}} = N_s \end{aligned} \tag{14}$$

GENERALIZATION

Since the wave equation is linear, we can obtain the wave field generated by many discrete sources located in x_{0i}, y_{0i}, z_{0i}:

$$\psi(t,x,y,z)=\sum_{i=1}^{n}\frac{\psi_{0i}}{\sqrt{(x-x_{0i})^2+(y-y_{0i})^2+(z-z_{0i})^2}}\cos\left(\omega_i t-k_i\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}+\phi_i\right) \quad (15)$$

or, if the sources are spread over an area *Ω* (can be 1, 2 or 3 dimensional) continuous space:

$$\begin{aligned}\psi(t,x,y,z)=\int_{\Omega}\frac{\psi_0(\xi,\eta,\zeta)}{\sqrt{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2}}\cdot\\ \cdot\cos\left(\omega(\xi,\eta,\zeta)t-k(\xi,\eta,\zeta)\sqrt{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2}+\phi(\xi,\eta,\zeta)\right)d\Omega\end{aligned} \quad (16)$$

The generalization may extend to three dimensional waves, the wave function is a three-dimensional vector. For now is not for this article.

ENGINEERING SOLUTIONS

To mitigate the noise prediction and sound intensity level engineers chose a simple function, possibly inspired by theoretical solutions, which is calibrated, then using a wide range of situations. For example, for a point source, [3] the following formula is used:

$$L_{eq}=L_W-20\lg r-11 \quad (17)$$

where L_{eq} is the noise level of the source, L_w is the source power, and *r* is the distance. Following [11], a formula for an industrial source complex:

$$L_{eq}=L_i+10\lg S-20\lg r-14 \quad (18)$$

where L_i is the inner power of the source, and *S* is its outer surface. In general such formulas for different types of sources, they have the form:

$$L_{eq}=A-B\lg r \quad (19)$$

with constants *A* and *B*. We used such a formula in this paper, for theoretical and empirical modeling of a source. More complicated formulas are used, for example, in [2, 11].

NUMERICAL APPLICATION OF NOISE FOR THE MECHANICAL MOWERS

To test and understand how theoretical and engineering solutions will apply these solutions to simulate noise from mechanical mowers. These devices produce a very intense noise and therefore must be tested in order to establish a continuous working time limit for the user but also for those accidentally found further around the area of operation.

In particular, a source of theoretical monochromatic sound, such as (2), with which we try to mathematically model a mower noise emission as in Fig. 1. Noise measurements were made at different distances from the camera, in open terrain, without barriers, except some negligible dimensions. The mower type was Rx8 RURIS (Fig. 1). Measured noise level at 50 cm above the mowers engine, showed the following values of noise intensity estimators: L_{eq} = 95.1 dB. The frequency at which that signal was maximum is 250 hz.

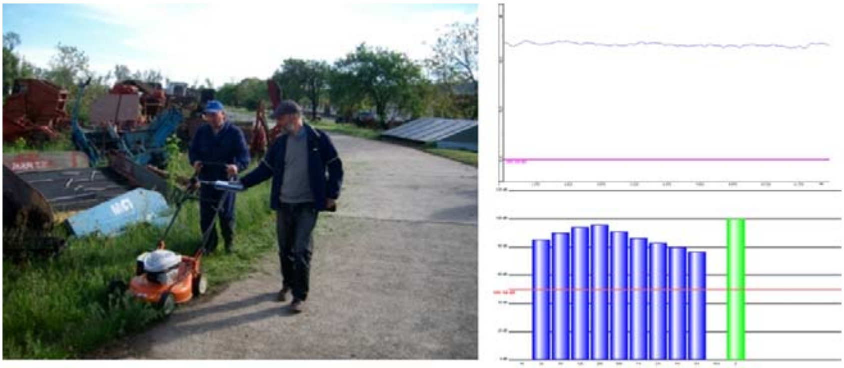


Figure 1. Mechanical mower during noise measurements

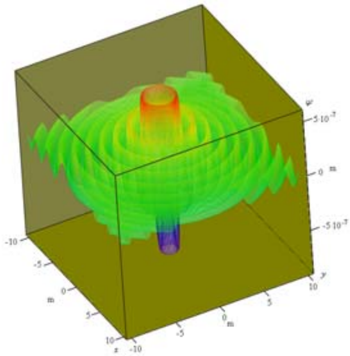
Through measurements, the complete attenuation of mowers noise is made at a distance of 51.79 m, which is equal to the background noise, 56.7 dB. Sound intensity level attenuates at background noise level at 39,725 m, while the noise level L_{eq} calculated by calibration at 41.58 m. It is noted that the model had no background noise.

For theoretical and empirical modeling of noise variations with distance at mowers, we used a type (19) formula:

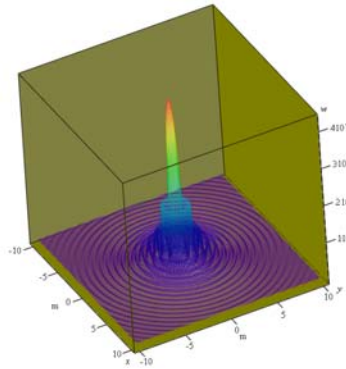
$$L_{eq} = 89.079 - 20\lg r \tag{20}$$

Formula (20) will be used for theoretical and empirical modeling of this type of mower. Likewise formulas were developed for other sources of agricultural noise pollution equipment: harvesters, mowers self-worn, mills for grinding grain, etc.

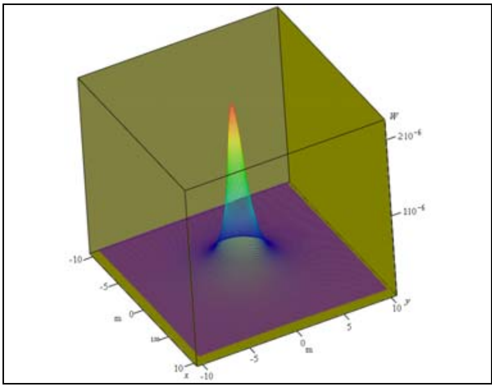
Using this source and measuring data from a distance of 0.5 m from the source, the theoretical solution is calibrated by choosing the constant ψ_0 to teke the value 0.00000116275 m^2, for a frequency of 250 Hz. It was considered that c = 343.2 m·s^{-1} and ρ = 1.29 kg·m^{-3}. The obtained graphics data is in Fig. 2, the theoretical and graphical solution in Fig. 3 Theoretical and empirical solution and theoretical (dotted curve in Fig. 3, b). A possible interpretation for the constant of the solution (2) for the wave equation is that it is outside the scope of the power, performance similar to that of constant S from the theoretical and empirical formula (18).



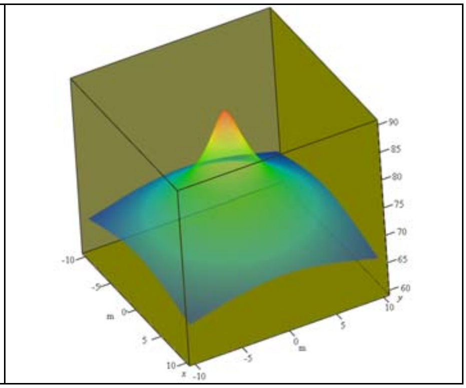
a) Acoustic wave elongation



b) Acoustic wave energy

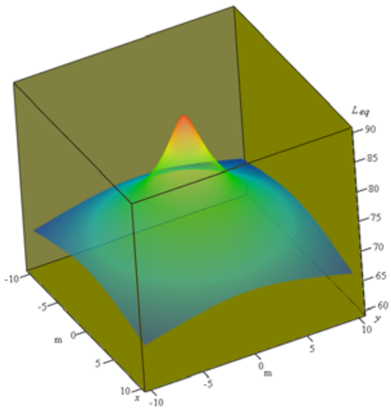


c) Average acoustic wave energy

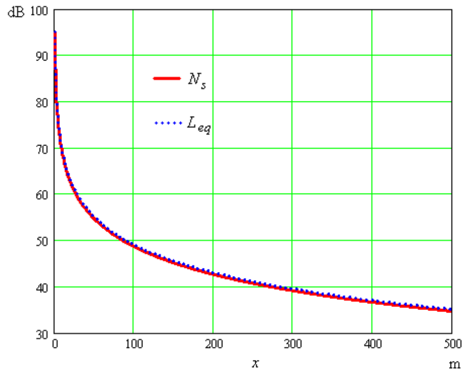


d) Sound intensity level (formula (10))

Figure. 2 Graphical representation of the main parameters a monochromatic spherical wave



a) Sound noise level formula (20)



b) Comparison between the intensity of sound

Figure 3. Noise level from the generated acoustic wave and comparison between the intensity of sound and noise. The continuous curve is calculated from equation (10), and the dotted, using formula (20)

CONCLUSIONS, POSSIBLE APPLICATIONS

The study presented in this article shows that the theoretical and theoretical-empirical solutions converge. In addition, it looks like many from the estimators of sound intensity or noise expresses the same result, a sound intensity level.

A complete theoretical solution is logical, but solving the wave equation for a wide variety of sources and, especially for a multitude of obstacles with different properties (absorption, reflection, refraction), is difficult to obtain, if not impossible. So it seems that the engineering solution of choice for empirical formulas for types of sources and types of obstacles is for now, the best solution. This solution is based on all major software products that are on the market at this time: Predictor-Lima (Bruel & Kjaer), Encustica (Canarina Environmental Software), SoundPLAN, etc.

Among the possible applications of the theoretical proposed solutions are:

- simulation of acoustic fields produced by a series (small in number) of discrete sources, placed on a continuous structure, without requiring any complex measurement;
- theoretical solutions also allow calculation of quantities that engineering tool do not give: acoustic pressure, acoustic wave composition effects.

Problems in solving fundamental problems such as absorption, reflection and refraction of waves still remain, of course, for now. Also finding theoretical and engineering solutions that have singularities is still a problem. For engineering solutions in [1] already a solution was proposed.

The theoretical solutions can be applied to configurations composed of a small number of sources in specific situations agriculture: agricultural machinery field configurations, mills additional nearby equipment, stationary or semi-stationary agricultural machines located near inhabited areas or nature reserves (which have a negative influence on wildlife), etc.

As spherical wave expression, formulas (17), (18), (19), have the singularity disadvantage in the point location source. Alternatives at singularities of theoretical-empirical formulas (17), (18), (19), were given in [1].

Another problem of theoretical and empirical formula type is a dimensional one. This problem is due to the *r* variables appearance, which has the dimension of a length, directly under the decimal logarithm. Then the logarithm of this quantity has no physical dimension with the correct meaning. In [1] we gave a solution that solves this problem.

BIBLIOGRAPHY

[1] Cârdei, P., 2008, Theory and simulation of the acoustic noise propagation. *9th WSEAS International Conference on acoustics & music: Theory & applications (AMTA ‚08)*, p. 47-49.

[2] LIMA Type 7812 version V.1, Technical Documentation, *Program System Manual*, Bruel & Kjaer.

[3] CANARINA Environmental Software, Canary Islands, Santa Cruz de Tenerife, Spain. Available through: http://www.canarina.com/acousticcontrol.htm [last access 29.06.2014.]

[4] Navcon Engineering Network, Noise & Vibration Consultants, Theory and Applications of the Indoor Noise Module (VDI 3760), SoundPLAN Indoor Factory Noise Module, Available through: http://www.navcon.com [last access 29.06.2014.]

[5] Iacob, C., Gheorghiţă, Ş., Soare, M., Dragoş, L., 1980. *Mechanical Dictionary.* Scientific and Encyclopedic Publishing House, Bucharest.

[6] IEC 61772, 2009, Nuclear power plants - Control rooms - Application of visual display units (VDUs) .

[7] http://ro.wikipedia.org/wiki/Sonometru [last access 29.06.2014.]

[8] http://users.utcluj.ro/~mbirlea/e/05e.htm [last access 29.06.2014.]

[9] http://en.wikipedia.org/wiki/Wave_equation [last access 29.06.2014.]

[10] Cârdei, P. 2008. Some proposal for the improvement of the simulations of the acoustic noise propagation. *International Journal of Mathematical Models and Methods in Applied Sciences*, Isuue 2, Volume 2, p. 285-293.

[11] Official Journal of the European Communities, DIRECTIVE 2002/49/EC OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL relating to the assessment and management of environmental noise, 2.

MATEMATIČKO MODELIRANJE I PRIMENA U AKUSTIČNIM APLIKACIJAMA U POLJOPRIVREDI

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Sažetak: U radu je predstavljeno ispitivanje i poređenje metoda modeliranja za akustične izvore, poređene su performance modela i predstavljene moguće aplikacije.

Predstavljeni i poređeni su sledeći matematički modeli: prirodni i teorijski model koji se zasniva na teorijskom i empirijskom modelu jednačine zvučnih talasa prostorne varijacije buke. Modeli su poređeni prema nivou intenziteta zvuka (koji je računat po egzaktnim formulama iz rešenja funkcije talasa jednačine zvučnih talasa) i nivoa buke dobijenih teorijskim i empirijskim modelom. Pokazano je da u najvećem broju slučajeva rezultati oba modela moraju da se kalibrišu da bi se dobili rezultati koji se slažu sa stvarnim vrednostima. Konačno, uporedili smo i poljske aplikacije oba modela.

Ključne reči: *akustični teorijski model, teorijsko-empirijski model, aplikacije, poređenja*

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