THE MATHEMATICAL MODEL OF CARROT SLICES DRYING

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Abstract: Drying of carrots slices was investigated in a laboratory dryer with a heater with thermostat. Carrots were grated in slices with a thickness of 1-2 mm, width of 4-5 mm and placed in the perforated container with layer thickness 30-35 mm. Based on the experimental data, the time dependency of moisture content and drying rate were calculated and presented. Using drying rate, the layer of carrot slices drying was simulated and receiving results compared with experimental data. Obtained measurement results are in high correlation with calculations. Presented mathematical model contains system of partial differential equations including the matter and environmental temperatures ($\Theta(x,t)$ and $T(x,t)$) and mass exchange ($W(x,t)$, $d(x,t)$) and it can be used for thick, porous medium layers, containing small particles, drying. Experimental and theoretical results showed that the carrot slices 3.5 cm thick layer was dry in 8 hours using a convective airflow with temperature 37 ^{o}C

Key words: drying, carrot, mathematical modeling

INTRODUCTION

Carrot is one of the most common used vegetable for human nutrition due to high vitamin and fiber content. Carrots have a variety of health effects, which are rich in vitamins, sugar, starch, potassium, calcium, phosphorus, iron and other nutrients and inorganic salts and 5 kinds of essential amino acids. Since higher temperature causes wilt and have a poor appearance on the carrots, refrigeration and controlled atmosphere storage has been used.

Another way for storage is to dried carrots and then stored. Drying process is the use of products with low water activity, thereby inhibiting the production of microbial

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reproduction and enzyme activity, and can give the flavor of a good product to achieve long-term storage, easy to transport, easy to consumer spending. During drying, heat is supplied and the volatile component, mainly water, is eliminated from the material mixture. During convective drying, two entirely different processes take place:
- elimination of water from the surface by warm air
- diffusion of water from within towards the surface due to the concentration difference.

Many studies were done to process carrot by air drying [1], sun drying [2] and solar drying [3]. Several researches have been done to the influence of some process parameters (temperature, sample thickness, air flow rate, etc). The effect of carrot slices on the drying kinetics was studied in [4]. The modeling of carrot cubes was made in [5].

The author studied influence of air-flow rates and effect of temperature to drying curve for carrot cubes. Liu Zhenghuai [6] studied the drying process of thick carrot slices heat transfer simulation, integrated heat diffusion process, slices carrots proposed internal heat transfer model and internal mass transfer model, the use of the third heat transfer boundary conditions were simulated and experimental compare done.

The aim of this research was to investigate thin carrot slices drying using small heated air and determining drying coefficient. This paper presents mathematical model for carrot slices layer drying.

MATERIAL AND METHODS

Carrots were grated in slices with a thickness of 1-2 mm, width of 4-5 mm. The carrot slices was placed in the perforated container with layer thickness of 30-35 mm (Fig.1).

Figure1. Sample of carrot slices

Was manufactured equipment, which allows to study the drying process of carrot slices (Fig. 2). Thermostat allows you to maintain a constant temperature of the sample

layer on one side. Inlet air temperature was 37 $^{\circ}$C. Heated air flow moving by convection. The container was weighted with electronic instruments to determine the quantity of water runoff, during the experiment.
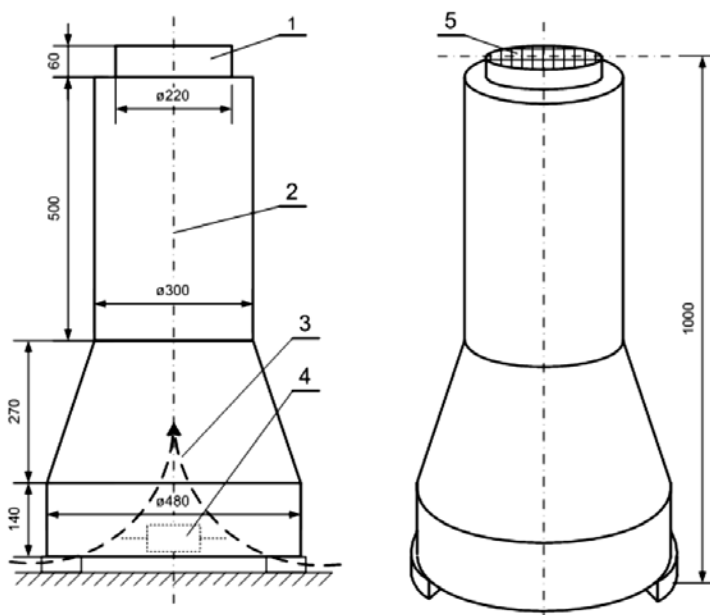

Figure 2. Experimental device scheme (1-outlet air temperature, 2- inlet air temperature, 3- inlet air, 4- heater with thermostat, 5- sieve with products)

Drying of any substance is based on heat-mass transfer processes. In our situation it is based on heat-mass transfer between carrots slices and inter-slices space. Since the slices thickness is very small, the internal diffusion in the drying process can be ignored.

We propose mathematical model which contain temperature and moisture functions of the matter (carrot slices) and inter-matter space (air). To describe the kinetics of drying process we assume the following:

- water evaporation in slices of carrot proceeds according to Dalton law,
- water is liquid in carrot,
- heat transfer between matter and drying agent (air) goes on by convection,
- the air flow takes place due to convection and it velocity is constant in layer of matter,
- inner temperature gradient for single matter slice is very small and has not been considered.

The heat-mass transfer model is based on laws of physics, i.e. the mass transfer law between matter and drying agent, the law of substance conservation, the law of heat transfer between matter and air and law of energy conservation. We obtained the following system of partial differential equations including the matter and environmental temperatures ($\Theta(x,t)$ and $T(x,t)$) and mass exchange ($W(x,t)$, $d(x,t)$) [7]:

$$\frac{\partial W}{\partial t} = K\left(W_p - W\right), \ t > 0, \ x > 0 \tag{1}$$

$$\frac{\partial d}{\partial t} + a_1 \frac{\partial d}{\partial x} = \frac{K}{a_2}\left(W - W_p\right), \ t > 0, \ x > 0 \tag{2}$$

$$\frac{\partial \Theta}{\partial t} = c_1(T - \Theta) + c_2\left(W_p - W\right), \ t > 0, \ x > 0 \tag{3}$$

$$\frac{\partial T}{\partial t} + a_1 \frac{\partial T}{\partial x} = c_0(\Theta - T) \ \ t > 0, \ x > 0 \tag{4}$$

where *x, t* - variable of space and time.

There:

$$a_1 = 3600\nu, \ \ a_2 = \frac{\gamma_a \varepsilon}{10\gamma_m}, \ \ c_0 = \frac{\alpha_q}{m\gamma_a c_a}, \ \ c_1 = \frac{\alpha_q}{(m-1)\gamma_m c_m},$$

$$c_2 = \frac{Kr}{100c_g}, \ \ K = exp\left(20.95 - \frac{6942}{T+273}\right), \ \ \alpha_q = 12.6\frac{\lambda}{L^2} \ .$$

Notations are:

ν - air velocity , $m \cdot s^{-1}$;

γ_a, γ_m - capacity of weight / air, matter respectively/, $kg \cdot m^{-3}$;

c_a, c_m - heat of drying air and moist matter, $kJ \cdot kg^{-1}$;

r - latent heat for water evaporation, $kJ \cdot kg^{-1}$;

$\varepsilon = m/(1-m)$ (m- porosity of matter),
W_p-equilibrium moisture content, dry basis, %;
K - drying coefficient, h^{-1};
α_q -interphase heat exchange coefficient, $kJ \cdot m^{-2} h^{-1} \cdot {}^0 C^{-1}$;
λ - rate of matter heat transfer, $kJ \cdot m^{-1} h^{-1} \cdot {}^0 C^{-1}$;
2L –carrot slices thickness, m.

Equilibrium moisture content W_p was obtained using S.Henderson's modified equation in Forte's interpretation :

$$W_p = \left(-\frac{1}{5869}ln\left(1 - \frac{\varphi}{100}\right)(T+273)^{0.775}\right)^{\frac{(T+273)^{1.363}}{5203}},$$

ϕ -heated air relative humidity , %.

Initial and boundary conditions for the system (1) - (4) can be given in the following way:

$$T\big|_{t=0} = \Theta\big|_{t=0} = \Psi_1(x), \ W\big|_{t=0} = \Psi_2(x), \ d\big|_{t=0} = \Psi_3(x) \tag{5}$$

$$T\big|_{x=0} = \vartheta_1(t), \ d\big|_{x=0} = \vartheta_2(t)$$

Initial conditions for the system (1)-(4) are given as follows:

$$\Psi_1(x) = \Theta_s, \ \Psi_2(x) = W_s, \ \Psi_3(x) = d_s$$

where Θ_s (^oC) is matter and intermatter air temperature in layer; W_s, d_s (%) are matter moisture and intermatter air humidity in layer. For carrot chips drying we chose constant boundary conditions:

$$\vartheta_1(t) = T_r, \ \vartheta_2(t) = d_r$$

where T_r (^oC) and d_r (%) heated air temperature and humidity.

The system (1)-(4) with boundary and initial conditions (5) can be solved numerically by difference scheme using weights [7].

RESULTS AND DISCUSSION

As shown by experiments, drying coefficient K is not constant but depends on the temperature. As our case T_r is constant, then using the experimental data to determine K dependence of the drying time.

As we do not know the real carrot slices moisture W_s and equilibrium moisture Wp, we taken the experimental data rationing $u(t)$:

$$u(t) = \frac{W(t) - W_p}{W_s - W_p}.$$

It should be noted that in the case of $W(t)$ understand the whole layer of the mean integral moisture. Using equation (1) and condition (5) we expressed the drying coefficient $K(t)$:

$$K(t) = -\frac{\ln(u)}{t},$$

where t-drying time (min).

Using the processed data was obtained carrot slices drying coefficient, depending on the drying time:

$$K(t) = 300.434 \cdot 10^{-5} + 1.12 \cdot 10^{-5} \cdot t \tag{6}$$

with coefficient of determination $\eta^2 = 0.99$.

Using expression (6), obtained from the experimental data, we solved equation (1). The experimental and numerical results is shown in Fig.3
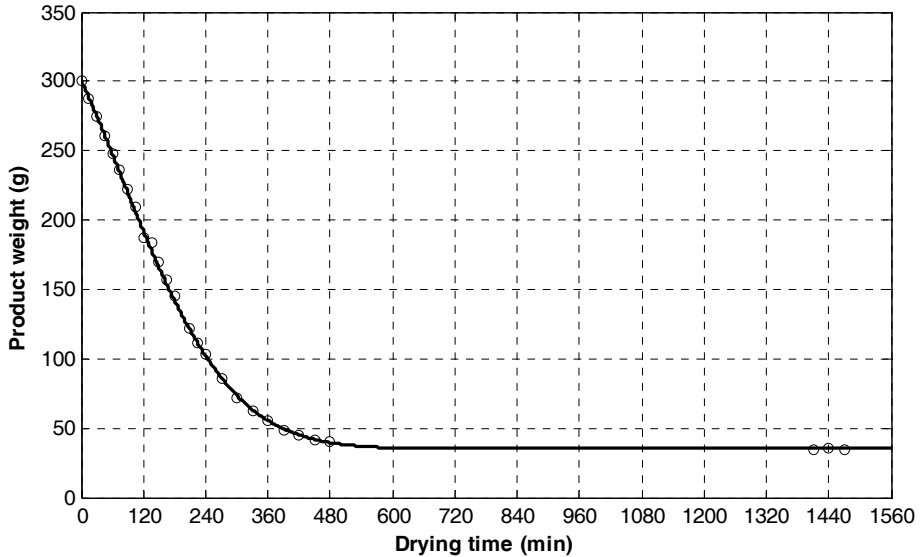

Figure 3. Carrot slices weight changes during the drying process
(ooo - experimental results, ---- - theoretical results)

The resulting drying coefficient expression (6) can be used for simulating a thick carrot slices layer drying using system (1)-(4) with initial and boundary conditions (5).

CONCLUSIONS

1. Experimental and theoretical results showed that the carrot slices 3.5 cm thick layer was dry in 8 hours using a convective airflow with temperature 37 $^{\circ}$C.
2. The offered mathematical model can be used for modeling the drying process of thin carrot slices in thick layer with drying agent velocity v, where are difference between air and matter temperatures. For thin layer we can use the first equation with changing drying coefficient.

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MATEMATIČKI MODEL SUŠENJA USITNJENE ŠARGAREPE

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Sažetak: Sušenje usitnjene šargarepe bilo je ispitivano u laboratorijskoj sušari sa grejačem sa termostatom. Šargarepa je usitnjena u rezance debljine 1-2 mm i širine 4-5 mm, a smeštena u perforirani sud, u sloj debljine 30-35 mm. Na osnovu eksperimentalnih podataka izračunate su i predstavljene zavisnosti sadržaja vlage i dinamike sušenja u funkciji vremena. Koristeći brzinu sušenja simulirano je sušenje sloja rezanaca šargarepe, a dobijeni rezultati su poređeni sa eksperimentalnim podacima. Dobijeni rezultati merenja su u visokoj korelaciji sa rezultatima proračuna. Predstavljeni matematički model sadrži sistem parcijalnih diferencijalnih jednačina koje uključuju temperature materijala i okoline ($\Theta(x,t)$ i $T(x,t)$) i razmenu mase ($W(x,t)$, $d(x,t)$). Može biti primenjen na sušenje debelih, srednje poroznih, slojeva koji se sastoje od malih rezanaca. Eksperimentalni i teorijski rezultati su pokazali da je sloj rezanaca šargarepe debljine 3.5 cm bio osušen za 8 časova, korišćenjem konvektivne vazdušne struje temperature 37 °C.

Ključne reči: sušenje, šargarepa, matematičko modeliranje