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WATER DROPLETS ANALYSIS: THE CLASSICAL AND QUANTUM HYDRODYNAMIC FRAMEWORKS

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Abstract: The design of a sprinkler irrigation system is always associated with a full understanding of the kinematics of the droplets during their aerial path. Resolving this problem involves both theoretical and experimental considerations. Among the theoretical studies the classical mechanical approach, based on the Newton's law, offers a useful tool to describe the trajectories of water droplets from the sprinkler nozzle to the ground. The problem becomes even more complicated when not just a single droplet alone is assessed but a multi-droplet system is accounted for. In addition to the inter-parameter dependencies, it is also observed an inter-droplet reciprocal repulsion, mainly due to electrical interactions between the hydrogen and the oxygen atoms of the different water molecules. An alternative to traditional classical procedures, to analyse water droplet dynamics in sprinkler irrigation, has been recently proposed in the form of a quantum approach. In this context, the whole classic-quantum and single-droplet versus multi-droplet alternatives need to be discussed and pinpointed and these are the main aims of the present paper which focuses on the theoretical part of the issue, thus highlighting the new perspectives of a deeper comprehension in the spray flow related phenomena. On the whole, the new approach leads to the concept of quantum trajectory in analogy to the well-established concept of classical trajectory and allows to recast the classical fluid dynamic equations into the so-called quantum equations.

Key words*: spray particles kinematics, single- and multi-droplet systems, classic and quantum mechanics, sprinkler water droplets, mathematical modelling*

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INTRODUCTION

Albeit the process of a liquid particle moving within a gaseous phase may describe many different technical applications, broadly investigated in many scientific sectors and from many different points of view, a complete, clear and generally applicable mathematical modelling is still far from having been achieved. Very recently Molle *et al*. [1] gave an extremely useful experimental contribution in the field of irrigation, which will be of substantial usefulness for future investigations and modelling attempts. The results of the present paper were mainly reached by featuring the sprinkler irrigation context. The outcomes, if suitably adapted, may also apply to other fields and aims, such as pesticides distribution, heat removal or fire suppression, to name but a few. The fundamental problem is not just that of solving the equations ruling the development of the system and of the phenomenon but, upstream of that, it lies in the characterization of such equations. According to the Authors of this paper such hurdle can be attributed to a not complete understanding of the system-process evolution. This of course should not be taken as a form of underestimation of the analytical difficulties due to the mutual interrelationships between the parameters that govern the process, on the one hand, and the characteristics of the particles involved, on the other. In fact the main thematic scientific literature [2-6] tries to overcome such computational complication binding the solutions achieved to the specific case studies faced. In general, however, the kinematic analysis of sprinkler water droplets during their aerial path is devised adopting a Newtonian approach and considering a single-droplet system. A ballistic form of the same viewpoint, based on a Newtonian approach for a single-droplet system, was also proposed by the Authors [7-9]. This approach will be described in the paper as a classic/single-droplet model. Coming again to the "not complete understanding of the system-process evolutions" quoted above, the issue has very recently led the Authors of the present work to the belief that to fully comprehend and describe the phenomenon another viewpoint could be considered: the quantum one [10]. The results which were arrived at [11] were in the form of the time-dependent Schroedinger's equation (TDSE) and of the Scale Relativity Theory [12] written as a Riccati equation. The former, in particular, was written for single-droplet systems, seen as waves and material particles [13] and considering a Lagrangian or Eulerian description for both steady and transient states. The present paper, therefore, will treat further possibilities to study the kinematic behaviour of both single- and multi-droplet systems during their aerial path in according to both the classic (i.e. Newtonian) and quantum approach. Moreover, a new procedure, the so-called Density Functional Theory for many-particle systems, has been highlighted, that allows to design a common 3-D space for the assessment of both the droplet trajectories and their interactions, and, so, to recast the TDSE into the quantum fluid-dynamic (continuity and Navier-Stokes) equations [14].

The basic picture that emerges is that of a multi-component fluid mixture moving into a 3-D space under the effects of common electric and magnetic fields and classic and quantum potentials.

THE CLASSIC MECHANICAL PICTURE

We do not wish to review the whole classic approach, as reported in the literature, but to explore the modelling possibilities in relation to the topic of the present paper. Anyway some more information on spray kinematics modelling (mainly Lagrangian) both in sprinkler irrigation and in chemical sprays contexts are also available [3,15-17], while spray drift Lagrangian modelling is treated in [18], and in [19]. Recently the Authors defined the following simplified analytical model feasible to solve water droplets kinematics, based on the Second Principle of Dynamics [7-9]:

$$\begin{cases} m\ddot{x} = -k\dot{x}^2 \\ m\ddot{y} = -k\dot{y}^2 - ng \end{cases} \tag{1}$$

developed in terms of parametric equations:

$$x(t) = \frac{m}{k}\ln\left(\frac{v_{0x}k}{m}t + 1\right) \tag{2}$$

$$\dot{x}(t) = \frac{mv_{0x}}{m + kv_{0x}t} \tag{3}$$

$$y(t) = h - \frac{m}{k}\ln\frac{\cos\left(\arctan\dfrac{\sqrt{\frac{k}{m}}v_{0y}}{\sqrt{\frac{n}{m}g}}\right)}{\cos\left(\arctan\dfrac{\sqrt{\frac{k}{m}}v_{0y}}{\sqrt{\frac{n}{m}g}} - t\dfrac{\sqrt{kng}}{m}\right)} \tag{4}$$

$$\dot{y}(t) = -\sqrt{\frac{ng}{k}}\tan\left[-\frac{\sqrt{ngk}}{m}t + \arctan\left(\sqrt{\frac{k}{ng}}v_{0y}\right)\right] \tag{5}$$

where:

f	[-]	- friction factor according to Fanning [20],
g	[m·s^{-2}]	- gravity,
h	[m]	- initial y co-ordinate,
$k = \frac{f\rho A}{2}$	[kg·m^{-1}]	- friction coefficient,
m	[kg]	- particle mass,

n [kg] - droplet actual mass (buoyancy),
t [s] - time,
v_{0x}, v_{0y} [m·s^{-1}] - initial horizontal and vertical velocity components,
x, y [m] - co-ordinates along the horizontal and vertical axes,
$\dot{x}$, $\dot{y}$ [m·s^{-1}] - velocities along the horizontal and vertical axes,
$\ddot{x}$, $\ddot{y}$ [m·s^{-2}] - accelerations along the horizontal and vertical axes.

Being the model analytical, albeit simplified, it is applicable to a variety of problems but the more reliable results were obtained for high Reynolds numbers. Obviously, as mentioned above, the model presented is one of the possible ones which can describe a single-droplet system from a classic viewpoint: the choice was mainly due to the fact that such model is tightly related to the second law of dynamics, as previously mentioned. To complete the topic, anyway, one may in general face the kinematic analysis of a multi-droplet system (i.e. composed of N droplets) from a classic viewpoint by means of the following analytical expression [21]:

$$m_k \frac{d^2 Q_k(t)}{dt^2} = -\nabla_k V \cdot Q(t) \tag{6}$$

where:
m_k [kg] - k-th particle mass ($1 \le k \le N$),
Q [m] - classic trajectory,
V [kg·m·s^{-2}] - potential function accounting for time dependence,
∇_k [m^{-1}] - 3-D gradient operator referred to the k-th particle.

The classical procedure needs a quantitative approach to check how reliable the predictions are. In order to validate the kinematic model just reported, comparisons of field measurements and theoretical values have been already reported in literature [2, 6, 8]. In conclusion the model here defined proves to be kinematically reliable in its predictions from a qualitative and quantitative points of view, particularly when droplets having a "not too small" diameter are considered.

Quantum mechanics for a single particle

The discrepancies between a classic *Vs.* quantum description, on the one hand, and between a single-droplet *Vs.* multi-droplet one, on the other, may be highlighted examining the following expressions for single- and multi-droplet systems (respectively) as compared to those in the previous section of this paper [11, 21]:

$$m \frac{d^2 Q(t)}{dt^2} = F(t) \tag{7}$$

$$m_k \frac{d^2 Q_k(t)}{dt^2} = \nabla_k \left(V \mid V_{qu}^{\psi_t} \right) Q(t) \tag{8}$$

where:

F [N] - force,
$\hbar$ [J·s] - the Dirac constant,
$V_{qu}^{\psi_t} = -\sum_{j=1}^{N} \frac{\hbar^2}{2m_j} \frac{\nabla_j^2 |\psi|}{|\psi|}$ [kg·m·s^{-2}] - quantum potential, ($1 \le j < k \le N$).

Comparing Eq. (8) with Eq. (6), the first useful consideration is that, if the quantum potential tends to zero, then the quantum and classic kinematic pictures tend to coincide. But as a quantum viewpoint presumes that the "object" evaluated is not just a material particle but also a wave, then for each element of a multi-droplet system one may write the *TDSE* as:

$$D^2 \nabla^2 \psi(\vec{x},t) - \frac{1}{2} \cdot m \cdot V(\vec{x},t) \cdot \psi(\vec{x},t) = -i \cdot D \cdot \left(\frac{\partial}{\partial t} \right) \cdot \psi(\vec{x},t) \tag{9}$$

With:

$$\psi(\vec{x},t) = R(\vec{x},t) \cdot \exp[S(\vec{x},t)] \tag{10}$$

where:
D [m^2·s^{-1}] - diffusion coefficient,
i [-] - imaginary unit,
R [-] - wave amplitude,
S [rad] - wave phase.

Eq. (9) can be re-written in the form of continuity and Euler-type "quantum fluid-dynamic equations", respectively [11, 14, 22]:

$$\frac{\partial}{\partial t} \rho(\vec{x},t) + \nabla[\rho(\vec{x},t) \cdot \vec{v}(\vec{x},t)] = 0 \tag{11}$$

$$\frac{\partial}{\partial t} \vec{v}(\vec{x},t) \equiv \left[\frac{\partial}{\partial t} + \vec{v}(\vec{x},t) \cdot \nabla \right] \vec{v}(\vec{x},t) = -\frac{1}{m} \cdot \nabla[V(\vec{x},t) + Q(\vec{x},t)] \tag{12}$$

where:
ρ [kg·m^{-3}] - density,
$\vec{v}$ [m·s^{-1}] - velocity.

Eq. (9) can be re-worked by means of Nottale's Scale Relativity theory [12] using a probability density function for a semi-infinite domain [23] for writing the second law of dynamics in the complex field:

$$-\nabla u = m \cdot \frac{\partial}{\partial t} W \tag{13}$$

where:
u [m^2·kg·s^{-2}] - scalar potential,
W [m·s^{-1}] - complex velocity.

Dividing the real and imaginary parts in Eq. (13) (*U* is the imaginary part of *W*) one gets:

$$\begin{cases} -D\cdot\Delta U-(U\cdot\nabla)U=-\nabla u \\ \dfrac{\partial}{dt}U=0 \end{cases} \tag{14}$$

which (first equation) may be re-written for a 1-D path as a Riccati equation [24], being *c* a constant and *y(x)* an arbitrary function of *x*:

$$\frac{d}{dx}U(x)=-\frac{m}{\hbar}U^2(x)+\frac{2}{\hbar}\cdot\left(u(x)-c\cdot m\right) \tag{15}$$

$$\frac{d^2}{dx^2}y(x)=-\frac{2m}{\hbar^2}\cdot\left(u(x)-c\cdot m\right)\cdot y(x)=0 \tag{16}$$

Both are very powerful tools as they allow for quantum particles computations avoiding the *TDSE*, even if just for 1-D domains, which is useful in particular cases as for instance a droplet vertical downfall.

Quantum mechanics for many-particle systems

Considering multi-droplet systems the TDSE needs to be suitably re-written, provided that water has a V-shaped molecule resulting in a magnetic field due to the electric potential between oxygen and hydrogen. This results in [14]:

$$\left[\frac{1}{2}\sum\nolimits_j\left(-2iD\nabla_j-\vec{K}\left(\vec{x}_j,t\right)\right)^2+\frac{1}{m}V\left(\vec{x}^N,t\right)\right]\psi\left(\vec{x}^N,t\right)=2iD\frac{\partial\psi\left(\vec{x}^N,t\right)}{\partial t} \tag{17}$$

being the electric potential $V\left(\vec{x}^N,t\right)$:

$$V\left(\vec{x}^N,t\right)=\sum_{j=1}^{N}\left[V_o\vec{r}_j-e\Phi\left(\vec{x}_j,t\right)\right]+U\left(\vec{x}^N,t\right) \tag{18}$$

where:

$\vec{K}\left(\vec{x}_j,t\right)$	[V·T·m^{-3}]	- vector potential for the electromagnetic field,
$\Phi\left(\vec{x}_j,t\right)$	[V·C^{-1}]	- external time-dependent scalar potential,
$\vec{x}^N$	[x, y, z]	- *N*-particle coordinates,
e	[C]	- elementary charge,
$\vec{r}_j$	[m]	- location vector ,
V_o	[V·m^{-1}]	- starting potential

It is now possible to write the continuity equation:

$$\frac{\partial \rho^N(\vec{x}^N,t)}{\partial t}+\sum_{k=1}^{N}\nabla_k \vec{j}_k\left(\vec{x}^N,t\right)=0 \tag{19}$$

and the Euler-type equation:

$$\frac{\partial \vec{v}_k(\vec{x}^N,t)}{\partial t}+\sum_j\left(\vec{v}_j\left(\vec{x}^N,t\right)\nabla_k\right)\vec{v}_j(\vec{x}^N,t)+\sum_j\left(1-\delta_{jk}\right)\vec{v}_j(\vec{x}^N,t)\times\left(\nabla_k\times\right)\vec{v}_j(\vec{x}^N,t)=$$

$$=-\left(e\vec{E}(\vec{x}_k,t)+\frac{e}{c}\vec{v}_k(\vec{x}^N,t)\times\vec{B}\left(\vec{x}_k,t\right)\right)-\frac{1}{m}\nabla\left[V_0(\vec{x}^N,t)+U(\vec{x}^N,t)+Q(\vec{x}^N,t)\right] \tag{20}$$

where:

$\rho^N(\vec{x}^N,t)=R^2(\vec{x}^N,t)$ [m^{-3}] - N-particle density,

$\vec{j}_k\left(\vec{x}^N,t\right)=\rho^N(\vec{x}^N,t)\vec{v}_k(\vec{x}^N,t)$ [m^{-2}·s^{-1}] - flow-fluid density,

$\vec{v}_k(\vec{x}^N,t)=\frac{\hbar}{m}\nabla_k S(\vec{x}^N,t)-\frac{e}{mc}A(\vec{x}^N,t)$ [m·s^{-1}] - k-th particle field velocity,

$\vec{E}(\vec{x}_k,t)=-\nabla\Phi\left(\vec{x}_j,t\right)-\frac{1}{c}\cdot\frac{\partial A(\vec{x}^N,t)}{\partial t}$ [V·m^{-1}] - external electric field,

$\vec{B}\left(\vec{x}_k,t\right)=curlA(\vec{x}^N,t)$ [T] - external magnetic field.

$U(\vec{x}^N,t)$ [C^2·m^{-2}] - mutual inter-electrical Coulomb repulsion.

C [-] - numerical coefficient,

$A(\vec{x}^N,t)$ [V·s·m^{-1}] - vector potential of the classic forces,

$S(\vec{x}^N,t)$ [rad] - wave function phase.

The first integrations of Eq. (19) and Eq. (20) were carried out by Madelung [25], and the work was successively extended by Bohm [26, 27].

Quantum mechanics within a Density Functional Framework (DFF)

The quantum mechanics approach for many-particle systems leads to the continuity (15) and Eulero (16) equations in configuration space, thus involving the N-particle density $\rho^N(\vec{x}^N,t)$ with the 3N-D velocity field (corresponding to the k-th particle) given by:

$$\vec{v}_k(\vec{x}^N,t)=\left(\frac{\hbar}{m}\right)\nabla_k S(\vec{x}^N,t)-\left(\frac{e}{mc}\right)\vec{A}\left(\vec{x}_k,t\right) \tag{21}$$

The quantum mechanical equations are, however, appealing only if they are in 3-D space in terms of the basic variables $\rho(\vec{x},t)$ and $\vec{J}(\vec{x},t)$ and for N-particle systems, as a sprinkler spray flow, to obtain the continuity and Euler equations of quantum mechanics in 3-D space one can resort to the DFF which employs a partitioning of the particle-density and the current-density variables [14]. The DFF provides a single-particle based approach for the description of the motion of many-particle systems in 3-D space. In the context of the DFF, the single particle density and the current density (for the k-th particle trajectory)

are, respectively, given by $R_k{}^2(\vec{x}^N,t)$ and $\rho_k(\vec{x},t)\vec{v}_k(\vec{x},t)$, as explained in the previous chapter, with the corresponding velocity field now expressed as:

$$\vec{v}_k(\vec{x},t)=\left(\frac{\hbar}{m}\right)\nabla_k S(\vec{x},t)-\left(\frac{e}{mc}\right)\vec{A}_{eff}(\vec{x},t) \tag{22}$$

where:

$\vec{A}_{eff}(\vec{x},t)$ [V·s·m^{-1}] - effective classic forces potential.

Within this ground the continuity equation can be written as:

$$\frac{\partial\rho_k(\vec{x},t)}{\partial t}+\nabla\vec{j}_k(\vec{x},t)=0 \tag{23}$$

and the Euler equation as:

$$\frac{\partial\vec{v}_k(\vec{x},t)}{\partial t}=-\frac{e}{m}\left[\vec{E}_{eff}(\vec{x},t)+\frac{1}{c}\vec{v}_k(\vec{x},t)\times\vec{B}_{eff}(\vec{x},t)\right]-\frac{1}{m}\nabla\left[V_{eff}(\vec{x},t)+Q_k(\vec{x},t)\right] \tag{24}$$

where:

$\vec{E}_{eff}(\vec{x},t)=-\nabla\Phi(\vec{x},t)-\frac{1}{c}\frac{\partial}{\partial t}\vec{A}_{eff}(\vec{x},t)$ [V·m^{-1}] - effective electric field,

$\vec{B}_{eff}(\vec{x},t)=curl(\vec{A}_{eff}(\vec{x},t))$ [T] - effective magnetic field.

And the quantum potential can be expressed as:

$$Q_k(\vec{x},t)=\frac{\hbar^2}{2m}\nabla\rho_k(\vec{x},t)\frac{\nabla\rho_k(\vec{x},t)}{\rho^2{}_k(\vec{x},t)}-\frac{\hbar^2}{4m}\frac{\nabla^2\rho_k(\vec{x},t)}{\rho_k(\vec{x},t)} \tag{25}$$

which is trajectory dependent.

The Euler equation (22) can be recast into the Navier-Stokes equation given by [28]:

$$\frac{\partial\vec{j}_k(\vec{x},t)}{\partial t}=-\frac{e}{m}\left[\rho_k(\vec{x},t)\vec{E}_{eff}(\vec{x},t)+\frac{1}{c}\vec{j}_k(\vec{x},t)\times\vec{B}_{eff}(\vec{x},t)\right]-\frac{1}{m}\rho_k(\vec{x},t)\nabla V_{eff}(\vec{x},t)+\nabla\vec{T}_k(\vec{x},t) \tag{26}$$

where $\vec{T}_k(\vec{x},t)$ represents the stress tensor expressed as:

$$\vec{T}_k(\vec{x},t)=\left(\frac{\hbar}{2m}\right)^2\nabla\nabla\rho_k(\vec{x},t)+\frac{1}{\rho_k(\vec{x},t)}\left[\vec{j}_k(\vec{x},t)\vec{j}_k(\vec{x},t)-\left(\frac{\hbar}{2m}\right)^2\nabla\rho_k(\vec{x},t)\rho_k(\vec{x},t)\right] \tag{27}$$

The stress tensor is due to the contributions of both the quantum potential $\vec{Q}_k(\vec{x},t)$ and the current density of the *k*-th particle trajectory. The jet flow is featured as a mixture of N components (particles) and each component, described by Euler equation,

is characterized by common effective electric and magnetic fields, and by a trajectory-dependent quantum force of stress tensor [14].

For many-particle systems, as a sprinkler spray flow, the DFF represents a versatile tool for description of equilibrium as well as dynamical characteristics of the system. The basic picture is that of a multi-component fluid mixture moving in common effective electric and magnetic fields and component-specific quantum potentials.

This approach leads to the concept of quantum trajectory, in analogy to the well-established concept of classical trajectory and can represent an exciting area of research in sprinkler irrigation systems and, more generally, in the agricultural environment.

THE DYNAMICAL AND NUMERICAL APPROXIMATIONS

In any case an analytical "closed form" solution of the equations describing the quantum kinematics of particles is obviously extremely difficult and even the most advanced techniques often fail to achieve such purpose, albeit in the years to come this attempt will not be abandoned. This is why, recently, different forms of approximation have been introduced to treat the "quantum fluid-dynamic equations": among those, literature reports numerical and dynamical approximations [29], which are both currently being developed. The formers may rely on Eulerian, Lagrangian or Arbitrary Lagrangian-Eulerian descriptions, all characterised by advantages and disadvantages. Lagrangian descriptions are easier in the form through which they write down the equations, as the grid moves with the particle and follows its evolution; but they become difficult to handle as, step after step, the grid becomes non-uniform with problems in the accuracy of the flow solution. Eulerian descriptions are complicate at the beginning of the simulation, due to an increased analytical complication, but prove to be more practical afterwards as the grid does not change with time. A uniform grid following the flow evolution is instead met in the Arbitrary Lagrangian-Eulerian descriptions, also adopted in some computational fluid dynamics codes.

Within each of the three procedures, a given numerical approach can be further subdivided into different algorithms for evaluating derivatives and propagating in time such as the meshless Moving Least Squares (MLS) [29]. The MLS tends to average out any numerical error which may be accumulating in the solution ,helping by this means to stabilize the computational process. The advantages of Mesh based approaches include, also, computational efficiency, higher resolution, accuracy and stability.

The dynamical approximations do not rely in a mathematically-simplified description of the problem but in a physically-simplified one by superimposing some particular conditions (e.g. incompressible flow) or neglecting some other characteristics considered not so relevant to the whole picture. Several approximate methods have been developed in recent years, such as the Linearized Quantum Force (LQF), the Derivative Propagation Method (DPM) and the Vibrational Decoupling Scheme (VDS) [22]. Obviously it would not be inconceivable to imagine a mixed numerical-dynamical approximation approach and we feel that on this aspect research will invest a part of its future resources: in relation to this challenge one should highlight that quantum trajectories can be treated quite similarly to the classic ones when considering, for the particles treated, the suitable relations among the dynamic and the potential part of the problem.

CONCLUSIONS

Remarkable progress has recently been made in the development and application of quantum trajectories as a computational tool for solving the TDSE, which involves the time evolution of the wave function. In the Quantum Theory of Motion (QTM) the complete description of a physical system needs the simultaneous presence of the "wave" and the "particle".

The wave motion is governed by the TDSE, and the motion of a particle guided by that wave, for a given initial position, is characterized by a velocity defined as the gradient of the phase of the wave function. An assembly of initial positions will constitute an ensemble of particle motions (the so-called quantum trajectories or Bohmian trajectories), guided by the same wave, and the probability of having the particle in a given region of space at a given time is provided by the quantum mechanical TD probability density [30].

A crucial link between QTM and Quantum Fluid Dynamics (QFD) is the quantum potential. In QTM, the particles are under the stress of forces originated from both classical and quantum potentials, while in QFD the fluid motion takes place under the influence of the external classical potential augmented by the quantum potential.

In addition to featuring water droplet ballistics in a sprinkler spray flow, novel quantum trajectory methods are being developed for a broad range of dynamical problems such as mixed classical-quantum dynamics density matrix evaluation in dissipative systems, and electronic non-adiabatic dynamics.

In this context, the present investigation starts from recent hypothesis made by the same Authors of this paper: a water droplet could be treated as a quantum object, characterised both by material particle and by wave properties. Thus the TDSE may be employed to study the process and a parallel classic-quantum description may be achieved, both for single-droplet and for multi-droplet systems. The latter systems are not only affected by the usual fluid-dynamic parameters but the mutual repulsions and attractions between particles are to be accounted for, in the form of electric-magnetic potentials bound to the molecular structure of water: this allows one to re-write the TDSE and the so-called "quantum fluid-dynamic equations" in a novel and more complete form. Future studies will deepen the novel modelling approach suggested to make it more and more suitable for practical applications.

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ANALIZA KAPLJICA VODE: KLASIČNI I KVANTNI HIDRODINAMIČKI OKVIRI

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Sažetak: Konstrukcija sistema za navodnjavanje sa rasprskivačima uvek je praćena potpunim razumevanjem kinematike kretanja kapljica kroz vazduh. Rešavanje ovog problema uključuje, kako teorijska, tako i eksperimentalna razmatranja. Među teorijskim studijama, klasični mehanički pristup, zasnovan na Njutnovim zakonima, daje korisno sredstvo za opis putanja kretanja kapljica vode od rasprskivača do tla. Problem postaje još komplikovaniji kad se ne posmatra samo jedna sama kapljica, već system više kapljica. Pored inter-parametarskih zavisnosti, proučavano je i među-kapljično uzajamno odbijanje, najviše usled električnih interakcija između atoma vodonika i kiseonika iz različitih molekula vode. Alternativa tradicionalnim klasičnim postupcima za analizu dinamike vodenih kapljica pri rasprskavanju, nedavno je ponuđena u obliku kvantnog pristupa. U ovom kontekstu treba proučiti ceo sistem klasičnog i kvantnog pristupa kao i alternative jedno-kapljične i više-kapljične strukture, pa su ovo osnovni ciljevi predstavljenog rada, koji se fokusira na teorijski deo problema, čime naglašava nove perspektive i celovitije obuhvata fenomen toka rasprsnutog spreja. Uopšte, novi pristup vodi do koncepta kvantne putanje u analogiji sa dobro ustanovljenim konceptom klasične trajektorije i dozvoljava da se prerade klasične jednačine dinamike fluida u tzv. kvantne jednačine.

Ključne reči*: kinematika čestica spreja, jedno- i više-kapljični sistemi, klasična i kvantna mehanika, kapljice rasprsnute vode, matematičko modeliranje*

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