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MODELING OF THE SOIL-RIGID WHEEL INTERACTION USING DISCRETE ELEMENT METHOD

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Abstract: In this paper the soil-rigid wheel interaction was analyzed using Discrete Element Method (DEM). Three types of soil model were created with different mechanical properties. After that, the rigid wheel was simulated in two different ways: in the first case it was created as a rigid wall and then as a rigid particle. In each calculation the sinkage of the wheel was measured under different vertical loads. After that, the results of the simulations were compared to the theoretical values. To determine the theoretical sinkage values the Bekker-formula was used. Finally the accuracy and the calculation times of the two simulation methods were compared to each other.

Key words: *DEM, soil, wheel, sinkage, Bekker-formula*

INTRODUCTION

The agricultural machines are come into contact with the soil by their wheels. Therefore the soil-wheel interaction is a very important phenomena which need to be investigated. In the previous century this interaction could be analyzed only by real field or laboratory tests, but the disadvantages were, that performing these tests was very expensive and requires a lot of time. As results some theories were born in the middle of the 20th century about the wheel's sinkage and the rolling resistance. These theoretical backgrounds were summarized by McKyes [1] and Sitkei [2] as well.

In addition the information technology has evolved a lot since the middle of the latest century. Numerical methods were developed as well to simulate the behavior of the materials under static or dynamic loading conditions. These simulations need a lot of

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computing time but nowadays it is possible to use these efficiently to simulate the materials. The best known is the Finite Element Method (FEM), but the granular assemblies can not be modeled with FEM because of the structure of the material. The soil consists of a bunch of very small soil-particles which slide and roll on each other during motion.

Therefore the most suitable numerical method to simulate the soil is the Discrete Element Method (DEM) which was published by Cundall and Strack [3]. In DEM the materials are modeled as a group of individual elements which have own displacements. The whole loading process is divided into small time steps and in each calculation cycle the particles' displacements can be determined according to Newton's second law. After that, the displacements at the next time step can be approximated using the so-called central differential method. During our work the PFC2D software was used where only non-deformable elements can be generated thus it is important to define the correct contact type between the elements. According to the previous researches [4-6] the Parallel Bond contact model was used to simulate the soil's cohesive behavior. This type of contact model was developed by Potyondy and Cundall [7] and was published in 2004.

There are two different ways to investigate the soil-wheel interaction with the PFC2D software. The wheel can be simulated as a wall element or in the other case it can be modeled as a rigid particle. In this paper these two simulation methods were compared to each other. In our earlier researches [8] three types of soil model were developed by calibrating the contact properties to the given soil's mechanical parameters (cohesion and internal friction angle). The wheel was pressed into these three soil models and during the simulations the sinkage of the wheel was measured. After that, the results were compared to the theoretical values which can be calculated using the so-called Bekker-formula.

MATERIAL AND METHODS

Theoretical background

In the 20th century Bekker was the first who investigated the soil-tire interaction. He found out that a tire with sufficiently high pressure acts as a rigid wheel and its sinkage can be calculated with Eq. 1 [9]:

$$z = \left[\frac{3 \cdot N}{(3-n) \cdot b \cdot k \cdot d^{0,5}} \right]^{\frac{2}{2 \cdot n+1}} \tag{1}$$

The meanings of the letters are summarized in Tab. 1. The n [-] and k [$Pa \cdot m^{-n}$] denote soil constant and stiffness constant, respectively and they depend on the quality of the soil and the width of the tire. The soil material can be described with its cohesion and internal friction angle values thus it will be very useful if the values of these constants can be attached only to the mechanical parameters of the material. So the k [$Pa \cdot m^{-n}$] stiffness parameter can be calculated as follows:

$$k = \frac{k_c}{b} + k_\phi \tag{2}$$

The k_c *[Pa·m$^{-(n-1)}$]* and k_ϕ *[Pa·m^{-n}]* are soil stiffness constants as well, but they are not depending on the wheel's geometry. In the book of McKyes [1] there is the Appendix 4, where the values of these constants can be found with the soil's mechanical parameters as well. From there, three types of soil were chosen as it is shown in Table 2. With these data the theoretical value of the wheel's sinkage with given geometry can be calculated.

Table 1. The meanings of the quantities in Bekker-formula

Quantity sign	*Unit*	*Description*
z	*m*	*Sinkage of the wheel*
N	*N*	*Load of the wheel*
n	-	*Soil constant*
b	*m*	*Width of the wheel*
d	*m*	*Diameter of the wheel*
k	$Pa\cdot m^{-n}$	*Soil stiffness constant*
k_c	$Pa\cdot m^{-(n-1)}$	*Soil stiffness constant*
k_ϕ	$Pa\cdot m^{-n}$	*Soil stiffness constant*

Table 2. The mechanical parameters and the stiffness-constants of the three soils [1]

Description / Stiffness constant	*Unit*	*Soil-type nr. 1*	*Soil-type nr. 2*	*Soil-type nr. 3*
Cohesion (c)	*kPa*	1.7	4.8	11.0
Internal friction angle (ϕ)	°	29.0	20.0	25.0
k_c	$Pa\cdot m^{-(n-1)}$	5.0	52.0	11.0
k_ϕ	$Pa\cdot m^{-n}$	1514.0	1127.0	1802.0
n	-	0.7	0.9	0.7

Settings of the simulations

To investigate the soil-tire interaction the soil model need to be created. Our purpose was to simulate three types of soil which are described with their mechanical properties in Tab. 2. In our earlier work [8], the contact parameters were calibrated to the chosen cohesion and internal friction angle values. In PFC 9 parameters have to be added to define the Parallel Bond contact model, these can be seen in Tab. 3. After that, a numerical direct shear simulation method was developed and many studies were performed. From the results of these tests the soil model's two mechanical properties can be calculated using the theory which was published by McKyes [1] and Sitkei [2] as well. The results are shown in the chapter Results and Discussion.

Table 3. The settings of the discrete element simulations [8]

Geometrical parameters				
Walls (box and rigid wheel)				
Length of the box	*mm*	*300.0*		
Height of the box	*mm*	*60.0*		
Width of the box	*mm*	*40.0*		
Diameter of the wheel	*mm*	*160.0*		
Width of the wheel	*mm*	*40.0*		
Balls				
Number of balls	-	*5000*		
Radius of the elements	*mm*	*0.66...1.5*		
Mechanical parameters				
Walls (box and rigid wheel)				
Normal stiffness	$N\cdot m^{-1}$	$1\cdot10^{20}$		
Shear stiffness	$N\cdot m^{-1}$	$1\cdot10^{20}$		
Balls				
Friction coefficient	-	*0.5*		
Density	$kg\cdot m^{-3}$	*1900*		
		Soil type nr. 1	*Soil type nr. 2*	*Soil type nr. 3*
Ball normal stiffness	$N\cdot m^{-1}$	$7\cdot10^{6}$	$4\cdot10^{6}$	$1\cdot10^{7}$
Ball shear stiffness	$N\cdot m^{-1}$	$7\cdot10^{6}$	$4\cdot10^{6}$	$1\cdot10^{7}$
Parallel Bond normal stiffness	$Pa\cdot m^{-1}$	$7\cdot10^{6}$	$4\cdot10^{6}$	$1\cdot10^{7}$
Parallel Bond shear stiffness	$Pa\cdot m^{-1}$	$7\cdot10^{6}$	$4\cdot10^{6}$	$1\cdot10^{7}$
Parallel Bond normal strength	*Pa*	$2\cdot10^{5}$	$1\cdot10^{5}$	$5\cdot10^{5}$
Parallel Bond shear strength	*Pa*	$2\cdot10^{5}$	$1\cdot10^{5}$	$5\cdot10^{5}$
Parallel Bond radius	-	*0.5*		

After the numerical direct shear tests a new box was created with 5000 particles. The calibrated contact parameters were added to the model and after that, one of the Otico's press wheel was assigned from [10] for further investigations. In the first case the wheel was modeled as a rigid wall and was compressed into the soil with different vertical loads in range of 30 to 300 N. At this type of simulations the so-called servomechanism had to be used to control the force of the wall-element. This calculation method can be found in the technical manual of the PFC2D software [11]. In every calculation cycle the wheel force can be calculated from the contact forces of the wall. After that, the vertical velocity of the wheel has to be adjusted to reach the given loading force. To calculate this velocity with Formula 4 the so-called gain parameter has to be determined from the contact stiffness values:

$$g = \frac{\alpha}{k_{wall} \cdot N_{contact} \cdot \Delta t} \tag{3}$$

$$v_{wheel} = g \cdot \left(F_{wheel} - F_{req}\right) \tag{4}$$

In Eq. 3, g *[-]* is the gain parameter, α *[-]* is the relaxation factor and was set to 0.5 to guarantee the stability of the calculation according to [11]. $N_{contact}$ *[-]* is the number of the contacts of the tire, k_{wall} *[N/m]* is the average stiffness of these contacts and Δt *[s]* denotes the value of the time step. F_{wheel} *[N]* and F_{req} *[N]* are the wheel-force at the given calculation cycle and the requested load of the tire, respectively. The simulations were stopped if the F_{wheel} force approximate to the F_{req} force with the accuracy of 0.5 %. In addition the geometrical and the mechanical properties of the simulations were shown in Tab. 3 and in Fig. 1 as well.

BACK OR FRONT GAUGE OR PRESS WHEELS MOUNTED STRAIGHT *(CAN BE ADAPTED ON A " " MACHINE)

tyre dimension	tyre reference	Tyre profile	Concept	Hub type	complete wheel reference
160x40 L	005176				

Figure 1. The geometrical dimensions of the Otico press wheel [10]

At the other type of the simulations the press wheel was simulated as a rigid particle. In case of these simulations the servomechanism was not necessary to use. Instead of that, the density *(ρ_{wheel} [kg·m^{-3}])* of the tire-element was calculated from the load and the volume of the wheel with Eq. 5:

$$\rho_{wheel} = \frac{F_{req}}{9.81 \cdot V_{wheel}} = \frac{F_{req}}{9.81 \cdot \left(\frac{d^2 \cdot \pi}{4} \cdot b \right)} \tag{5}$$

The meanings of the b *[m]* and d *[m]* parameters can be seen in Tab. 1 and the multiplier of 9.81 is necessary to calculate the wheel's weight in N-s from its mass in kg-s. After that, the gravity was added to the model to sink the tire into the soil. Finally the simulations were stopped if the velocity of the tire-element decreased under the value of 0.01 mm·s^{-1}. In Fig. 2 the Y position and the Y velocity of the tire and the wheel force were illustrated as green, blue and red line, respectively according to the number of calculation cycle. It can be seen clearly, that the value of the velocity and the position decreased at the start of the simulation. After the 2.75 E6th time steps the Y position of the tire and the wheel force are not changing, thus the wheel got into equilibrium state. So the sinkage of the wheel can be determined.

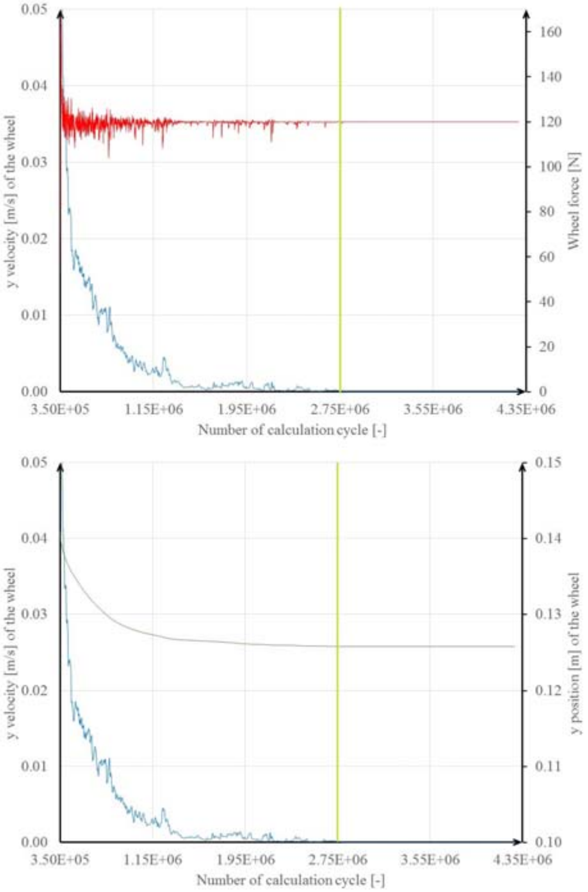

Figure 2. The changes of the Y position, Y velocity of the tire and the wheel force during the "ball-type" simulation in case of soil type nr. 1

During all simulations (the "wall-"and the "ball-type simulations" as well) the vertical position, the velocity of the wheel and the tire force were measured. The results can be seen in the next chapter.

RESULTS AND DISCUSSION

First the results of the numerical direct shear tests are presented. In Fig. 3 the 2D shear box can be seen in case of the soil type Nr. 1. The particles were represented as red and between them there are the tensile forces (the so-called parallel bond forces) as blue lines. The thicknesses of these lines are proportional to the magnitudes of the tensile forces. It can be seen that there are parallel bond forces only near the shear zone, so the simulations gave the same results as the real direct shear tests.

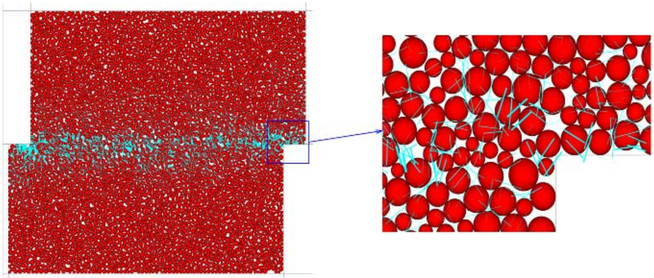

Figure 3. Results of numerical direct shear test in case of soil type nr. 1 [8]

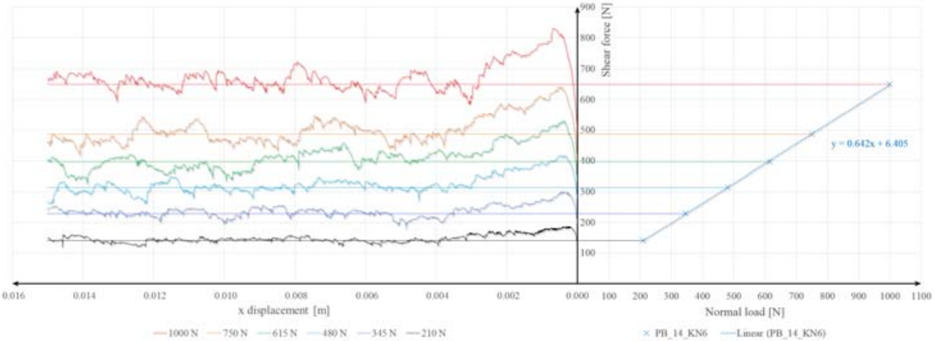

Figure 4. Results of the earlier researches in case of soil type nr. 1 [8]

On the left side of Fig. 4 the shear force was represented according to the shear displacements in case of each normal load (210 N, 345 N, 480 N, 615 N, 750 N and 1000 N, respectively). From a certain shear displacement value the shear forces were not changing sufficiently, so an average shear force value can be determined in case of each simulation. Illustrating these according to the normal loads, the so-called Coulomb-line can be drawn (see on the right side of Fig. 4). The soil's two mechanical properties can be calculated from the equation of these lines. The intersection of the vertical axis and the Coulomb-line defines the cohesion and the angle of the line and the horizontal axis defines the internal friction angle [1, 2].

Table 4. The calculated mechanical parameters of the three soil-model [8]

Description	*Unit*	*Soil type Nr. 1.*	*Soil type Nr. 2.*	*Soil type Nr. 3.*
Cohesion (c)	*kPa*	*1.78 (1.7)*	*5.09 (4.8)*	*7.52 (11.0)*
Internal friction angle	°	*32.70 (29.0)*	*31.84 (20.0)*	*31.84 (25.0)*
Relative error of the cohesion	*%*	*4.7*	*6.0*	*31.6*
Relative error of the internal friction angle	*%*	*12.8*	*59.2*	*27.4*

These calculations were performed in each three soil-model, the results can be seen in Tab. 4 (the chosen values from McKyes were represented in parentheses). In case of soil type nr. 1 the two mechanical parameters were calibrated well, but there were bigger errors of them in the other two cases. The details of these results were published in [8].

After calibrating the contact parameters to the soil's mechanical properties, the two types of the soil-wheel interaction simulations were performed. The results of them are shown in Figs. 5-9. In Fig. 5 the Y displacement of the rigid wheel was illustrated in case of vertical load of 120 N. On the left side the "ball-type" simulation, on the right side the "wall-type" simulation was represented. It can be seen that both of the two simulation methods gave the same results. The vertical displacement of the wheel and the soil's particles were very similar in both simulations. There was a difference as well, in case of the "ball-type" simulation the tire moved horizontally to the left a little bit therefore there were greater vertical displacements in the left side of the soil material (see the greater red-zone at the left side of Fig. 5).

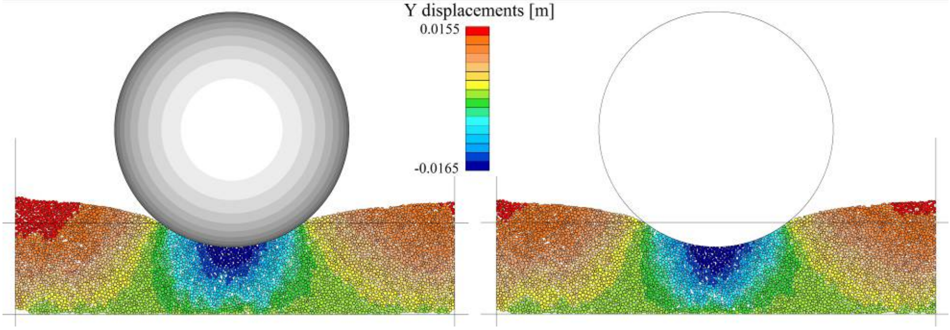

Figure 5. Results of the simulations in case of soil type nr. 1, vertical load of 120 N and "ball-type" simulation (left) and "wall-type" simulation (right)

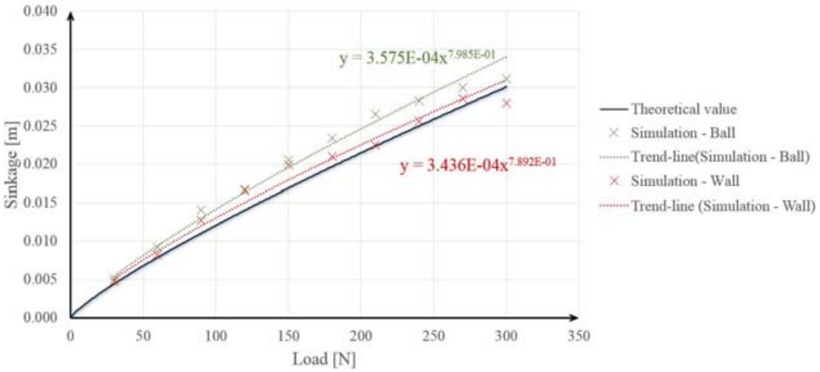

Figure 6. Results of the soil-wheel interaction simulations in case of soil type nr. 1

In Figs. 6-8 the sinkage values were represented in case of all soil types. The simulations were performed with 10 different vertical loads (30 N, 60 N, 90 N *etc.* up to 300 N with positive difference value of 30 N). The results were illustrated as *"X"* in the figures. A trend-line can be fitted to these points using the Wald-method in case of all soil-types. These trend-lines have to be compared to the theoretical line which can be drawn using Eq. 1. Our expectations were to get accurately results in case of soil type Nr. 1 because of the accurately calibration of the contact properties of the model. In Fig. 6 it can be seen that the tendency of the trend-lines are similar to the theoretical line.

In case of "wall-type" simulations the line follows closely the theoretical values, the maximum of the relative error in range of 30 N to 300 N vertical loads was 14.04 % (see Tab. 5). In case of "ball-type simulations" this value increased up to 22.44 %. At the other two types of soil model the same conclusion could be said. The most accurate results came in case of soil-type nr. 3 where the maximum relative error was 3.55 %.

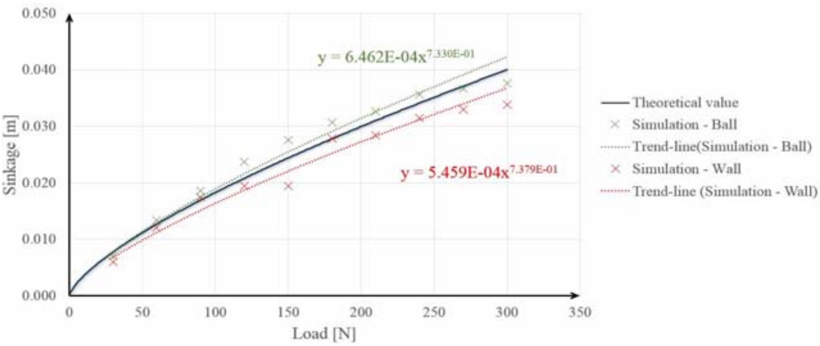

Figure 7. Results of the soil-wheel interaction simulations in case of soil type nr. 2

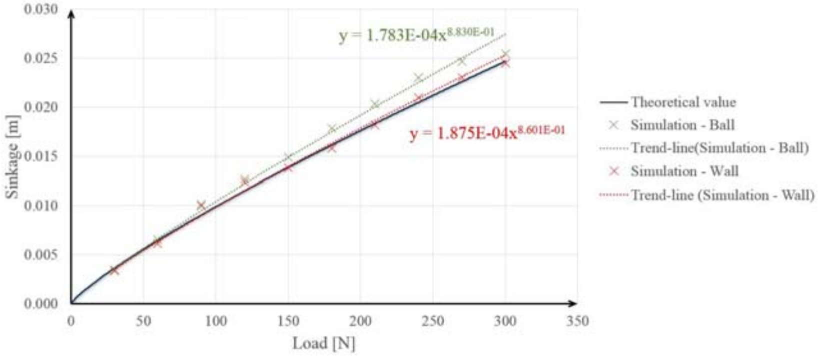

Figure 8. Results of the soil-wheel interaction simulations in case of soil type nr. 3

Table 5. The relative errors of the DEM simulations in range of 30 N to 300 N vertical loads

Description		*Unit*	*Soil type nr. 1.*	*Soil type nr. 2.*	*Soil type nr. 3.*
Relative error in "wall-type" simulations	*Min*	*%*	*3.03*	*8.16*	*0.00*
	Max		*14.04*	*13.03*	*3.55*
Relative error in "ball-type" simulations	*Min*	*%*	*13.01*	*1.25*	*0.07*
	Max		*22.44*	*5.72*	*11.18*

In addition it can be seen as well that the "ball-type" calculations always gave greater sinkage values as results than the "wall-type" simulations. Comparing the calculation times the "wall-type" simulations' need approximate 1.5 to 2.5 hours to calculate while the "ball-type" simulations had to run approximate 2.5 to 3 hours with the same computing background.

CONCLUSIONS

In this work the soil-rigid wheel interaction was investigated using the discrete element method. In our earlier publications three types of soil model were created, the parallel bond contact parameters were calibrated to the soil's mechanical properties. Using these soil models a tire was pressed into the soil with 10 different vertical loads and the sinkages of the wheel were determined.

In the numerical simulations first the tire was modeled as a rigid wall, after that as a rigid particle. Comparing the two simulation methods, the results show that the sinkage values from the "wall-type" calculations were less than in case of "ball-type" simulations in case of each soil model. In addition, if the calibration of the contact parameters is corresponding, the "wall-type" simulations gave more accurate results than the other. The wheel's sinkage values follow the theoretical values closely. The theoretical sinkages of the wheel were determined using the Bekker-formula. The maximum of the relative error was under 15 %.

Finally the time-consumptions of the two methods were compared to each other. The results show that the "wall-type" simulations needed less computing time than the "ball-type" simulations.

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MODELIRANJE INTERAKCIJE ZEMLJE I KRUTOG TOČKA METODOM DISKRETNIH ELEMENATA

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Sažetak: U ovom radu je je analizirana interakcija zemlje i krutog točka metodom diskretnih elemenata (DEM). Pripremljena su tri tipa zemljišta sa različitim mehaničkim osobinama. Posle toga, kruti točak je simuliran na dva različita načina: u prvom slučaju kao kruti zid, a zatim kao kruta čestica. U svakom proračunu mereno je propadanje točka pod različitim vertikalnim opterećenjima. Rezultati simulacija su poređeni sa teorijskim vrednostima. Za određivanje teorijskih vrednosti propadanja korišćena je Bekerova jednačina. Na kraju su međusobno poređene tačnost i vreme proračuna dva metoda simulacije.

Ključne reči: *DEM, zemljište, točak, propadanje, Bekerova jednačina*

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